INVESTIGATION OF LWR MODEL WITH FLUX FUNCTION DRIVEN BY RANDOM FREE FLOW SPEED

Jia Li[1], Qian-Yong Chen[2], Haizhong Wang[3], Daiheng Ni[4]

Department of Civil and Environmental Engineering[1,3,4]

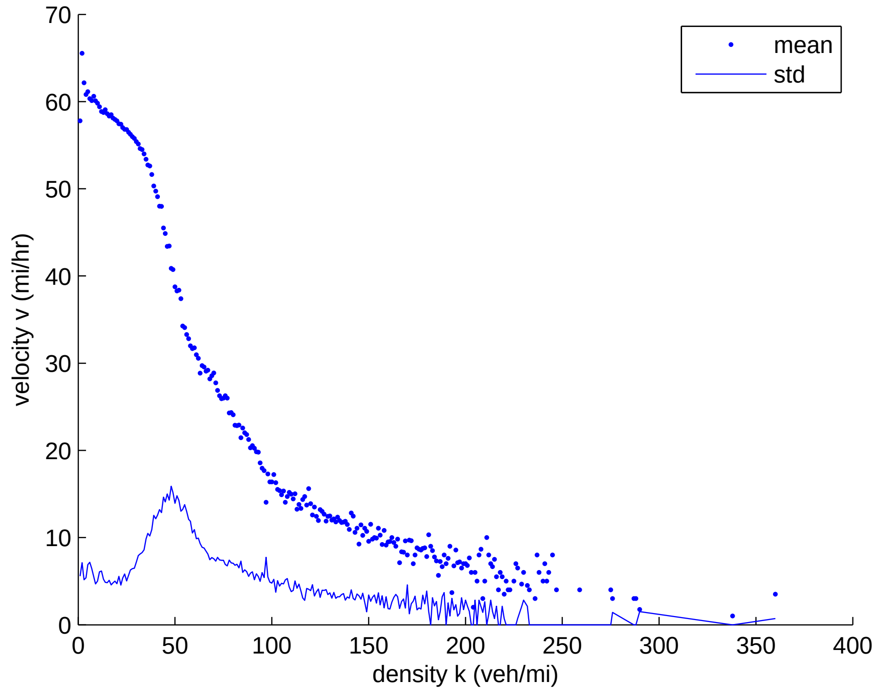
Department of Mathematics and Statistics[2]

University of Massachusetts Amherst

Outline

- Motivation of the study
  - interesting empirical observations
- Formulation of a new LWR model
  - incorporating the randomness
- Illustrative example of application
  - evaluation of predictability of traffic evolution
- Concluding remarks

MOTIVATION



First and second order speed-density relation at one typical site based on GA400 ITS data

Motivation

- Two types of randomness
  - statistical: ubiquitous and controllable
  - inherent: unique, reflecting the system dynamics
- Reflection of the LWR model
  - to account for the observed randomness
  - new insight into the operations of transportation system

Formulation: retrospect

- Conservation law:

$$\int_{\Omega_x} k(x,t)dx + \int_{\Omega_t} f(k(x,t))dt = 0$$

$$f(k(x,t)) = k(x,t)v(k(x,t))$$

FORMULATION: k-v CURVE

- Curve $\gamma : (k, v)$, parameterized by $0 \leq k \leq k_j$

$$(\frac{v(k)}{v_f})^{\alpha} + (\frac{k}{k_j})^{\beta} = 1$$

where $\alpha, \beta > 0$, interpolates $(0, v_f)$ and $(k_j, 0)$

FORMULATION: INTERPOLATION

- Brownian bridge

$$B^0(t) = B(t)|\{B(0) = 0, B(1) = 0\}$$

- A generalization

$$V^0(k) = V(k)|\{V(s) = \tilde{V}_s, s \in \mathbb{I}\}$$

Formulation: random v_f

- Define v_f a random process

$$v_f(k,\omega) : (\mathbb{R}^+, \Omega) \mapsto \mathbb{R}^+$$

In particular, assume

$$v_f(k,\omega) \overset{d}{\sim} \bar{v}_f + (sk + r)\epsilon$$

The flux function driven by v_f

$$f(k) \overset{d}{\sim} k(\bar{v}_f + (sk + r)\epsilon)(1 - (\frac{k}{k_j})^\beta)^{1/\alpha}$$

FORMULATION: MOMENTS

- Moment properties

$$E(v(k)) = \bar{v}_f(1 - (\frac{k}{k_j})^{\beta})^{1/\alpha}$$

$$Var(v(k)) = (sk + r)^2(1 - (\frac{k}{k_j})^{\beta})^{2/\alpha}$$

Formulation: decomposition

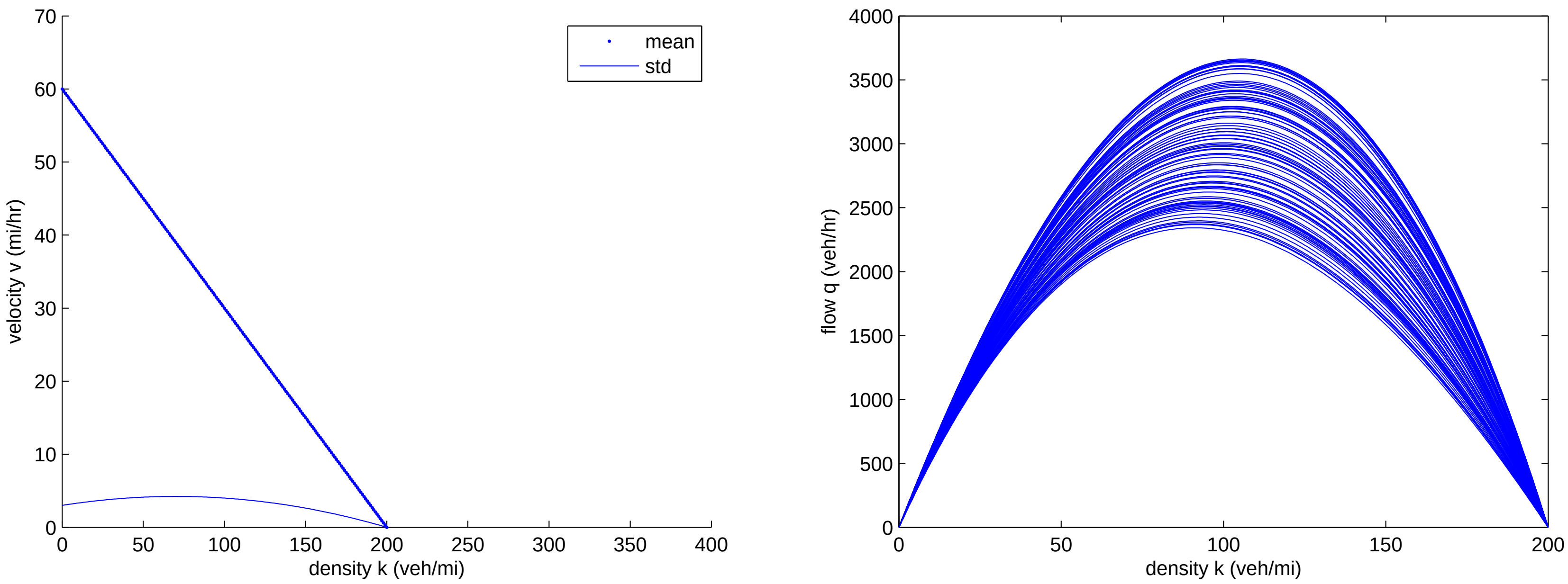
- Smoothness
- Boundedness

$$\sup_{0 \le k \le k_j} |f'(k)| \le 5k_j|\epsilon s| + 3|\epsilon r| + 3\bar{v}_f$$

- Decomposition

$$f(k) = f^+(k) + f^-(k)$$

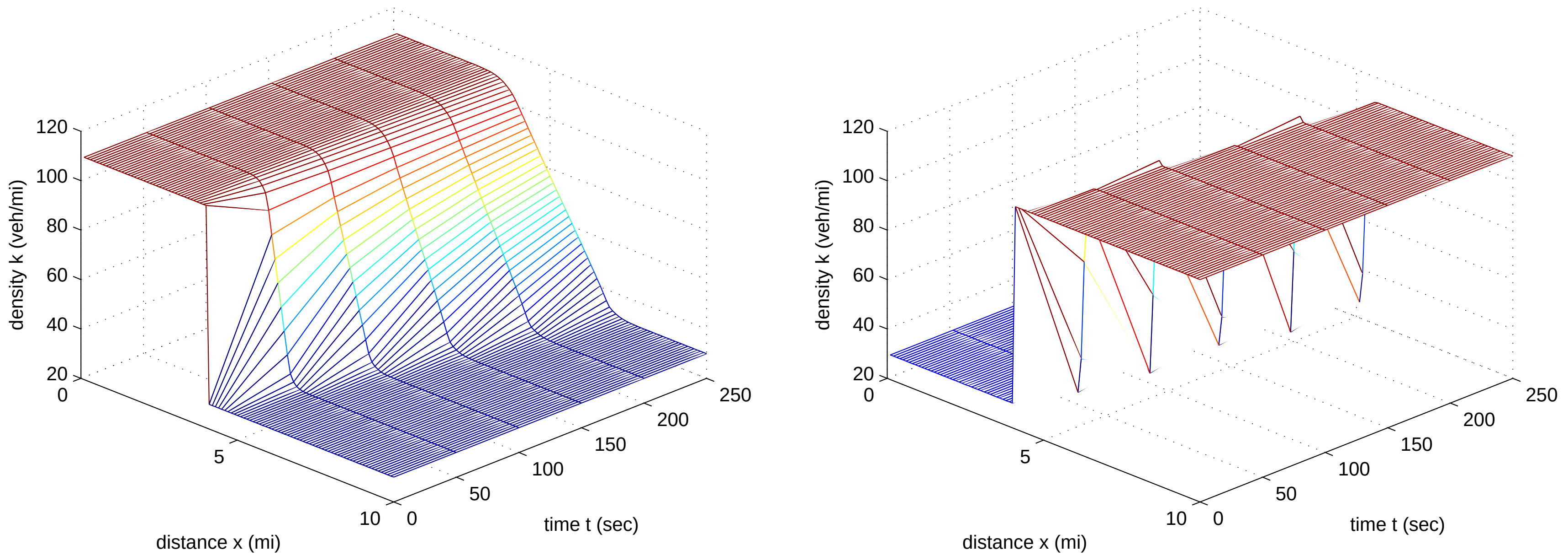
EXAMPLE: RANDOM FLUX FUNCTION



Hypothetical random speed-density relation (left panel) with $\alpha = 1$, $\beta = 1$, $s = 0.05$ and $r = 3$ and 100 realizations of flux function (right panel)

EXAMPLE: ENO-FD SCHEME

3rd order ENO (essentiallly non-oscillatory)-FD (finite difference)

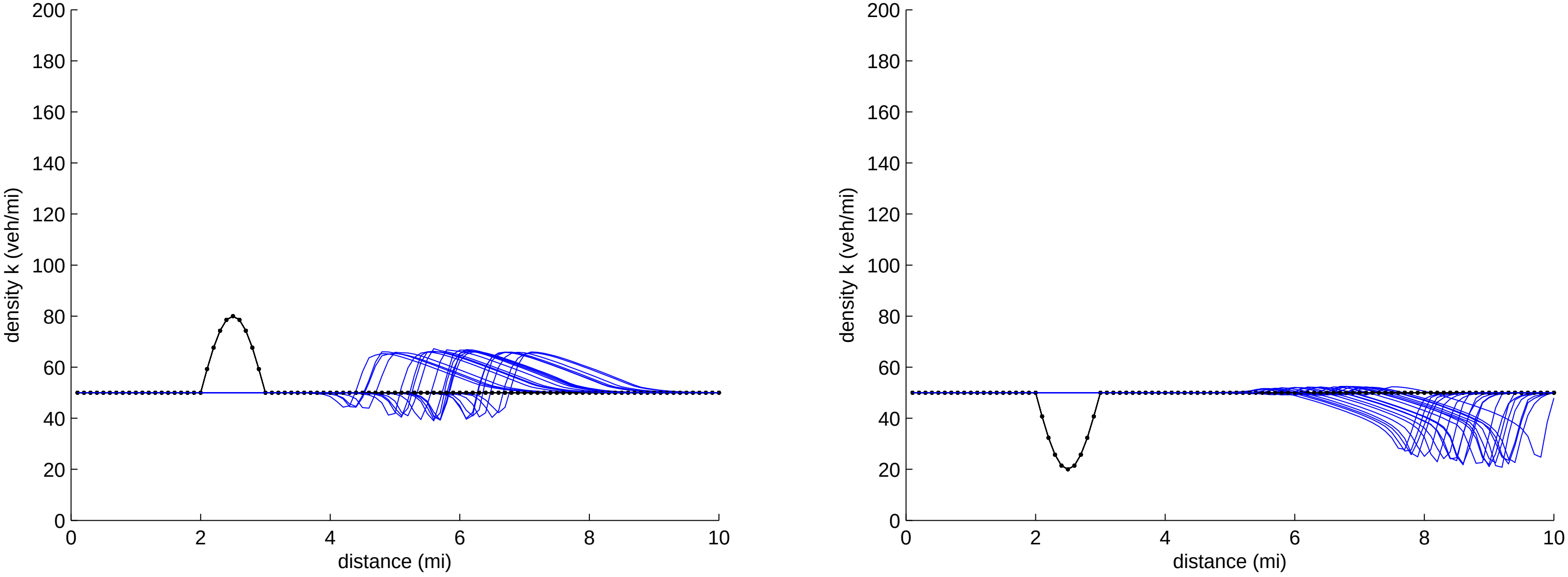


Temporal development of density k with $\epsilon = 0$: rarefaction waves (left panel) with $(k_l, k_r) = (110, 30)$, and shock waves (right panel) with $(k_l, k_r) = (30, 110)$

Example: sketch of algorithm

- Sketch of algorithm
  1. Generate ϵ, and obtain corresponding random flux function;
  2. Solve the LWR model using the ENO-FD;
  3. Go back to 1.

EXAMPLE: RESULT



Propagation of local disturbance (jam: left panel; vacuum: right panel) with random flux function, $t = 600$ sec, 20 realizations

EXAMPLE: IMPLICATION

- Quantities of interest

$$k_* = \max_{0 \le x_i \le 10} |k(x_i) - 50|, x_* = \text{argmax}_{x_i} |k(x_i) - 50|$$

- Coefficient of variation ($CoV = \sigma/\mu$)

	Mean of k_*	Std of k_*	Mean of x_*	Std of x_*
condition a	16.09	0.50	5.99	0.75
condition b	26.46	1.92	8.63	0.63

$$CoV_{a,k_*} > CoV_{b,k_*}, CoV_{a,x_*} > CoV_{b,x_*}$$

Concluding remarks

- Summary
  - Faithfulness, well-posedness and ease of solution
  - Randomness in the scope
- Gaps
  - Estimation
  - Temporal and spatial inhomogeneity