STRATEGIES TO IMPROVE DISSIPATION INTO DESTINATION NETWORKS USING MACROSCOPIC NETWORK FLOW MODELS

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ABSTRACT
Backups originating from destinations have been observed during evacuation. These backups usually occur due to congestion at the destination network, which result in spillbacks onto the evacuation routes. These spillbacks result in queuing and delays that hamper evacuation operations. This paper presents theoretical proofs for the fundamental flow/speed/concentration relationship and the speed-accumulation relationship (Greenshield's, Greenberg's and Bell-shaped model) at a network level. These relationships and, the relationships between inflow-accumulation and outflow-accumulation at a network level are studied using microscopic simulation. A strategy (Network Breathing Strategy) to improve dissipation of vehicles into the destination network is developed using these relationships between network level variables. A comparison of the network breathing strategy to a do-nothing strategy in a simulation network, showed a statistically significant increase in the number of vehicles dissipated into the network. This indicates that the application of such strategies on the destination networks would help improve evacuation operations, by clearing evacuation routes and reducing queuing.

Keywords: Evacuation, Network Level Models, Network Breathing, Evacuation Destination

INTRODUCTION

Recent natural disasters like hurricanes Katrina and Rita in 2005 have highlighted the importance for efficient transportation strategies, to ensure smooth and effective evacuation of people out of harms way. Strategies such as improving capacity by starting contraflow operations and demand staging have been widely studied and implemented as effective plans to reduce evacuation time.

During the South East U.S. Regional Transportation Analysis Meeting in 2000, it was observed that "More than half the evacuees felt like it took them more than five hours longer to reach their destination than they thought that it would" [1]. This is mainly due to the limited capacity of the exit ramps as well as congestion caused due to the large number of vehicles in the destination network. Usually evacuation routes terminate at large cities, and road networks in these cities are not designed to handle the large number of vehicles entering them during evacuation. This results in congestion which in turn leads to backups that extend for miles on the evacuation route. Though such phenomena have been observed repeatedly, limited literature on evacuation seems to have addressed this issue of network congestion at the termination node. Most simulation studies tend to assume ideal destinations, where vehicles leave the system as soon as they reach the destination irrespective of the number of vehicles already present in the destination road network. This provides a myopic perspective of analyzing evacuation routes. Therefore it is important to understand network level properties of traffic variables.

Initial attempts to understand relationships between network level variables consisted of Zahavi's [2, 3] work on the α-relationship between network level parameters of traffic intensity (I, the distance traveled per unit area), road density (R, length of road per unit area) and the weighted space mean speed (v). Using data from England and the U.S. he arrived at the relationship in equation 1.

$$I = \alpha R / v \tag{1}$$

Buckley and Wardrop [4] later showed that α was strongly correlated to the space mean speed. In a later field study, Ardekani [5] proved that the α parameter had a positive correlation to network concentration. This made the α parameter model highly inaccurate. Chapter 6 of the Traffic Flow Theory monograph revised 1997 [6] contains a comprehensive review of these macroscopic flow models.

In order to characterize flow of vehicles in urban network Prirgogine and Herman [7] proposed the two-fluid theory. The two-fluid model assumes that vehicular traffic in an urban network can be differentiated as stopped vehicles and running vehicles. These models were constructed between the average travel time per mile (T) versus the average running time per mile (T_r) using regression (equation 2). The parameters (k, T_m) involved in this two-fluid model were indicative of the quality of service of the networks.

$$T_r = T_m^{\frac{1}{k+1}} T^{\frac{k}{k+1}} \tag{2}$$

Mahmassani et al. [8] and Williams et al. [9] during their study of two fluid models using computer simulation showed that relationships between the three fundamental traffic variables speed/flow/concentration (equation 3) at a network level were similar to those on individual road facilities.

$$Q = KV \tag{3}$$

In a later simulation study Mahmassani et al. [10] found that both the linear V-K model proposed by Greenshield, and the non-linear 'bell-shaped' function proposed by Drake et al. [11] were able to describe the relationship between *V* and *K* fairly well. In their paper they also studied the effect of length and width of links as well as various traffic controls (perfectly coordinated, isolated and simultaneous signal operation) on the speed-concentration relationships and the flow-concentration relationships.

Even though these studies showed interesting results, due to the very few (six) data points used for the analysis, the conclusions in the paper are prone to major skepticism. Also each simulation run was done for constant concentration conditions, in which constant concentration was maintained by allowing vehicles to circulate in the network. Such concentration conditions generally do not prevail in real urban networks, where vehicles enter and leave, and concentrations in the network vary more dynamically.

Mahmassani et al. [12] conducted microscopic simulation experiments on larger urban networks than the ones studied in Mahmassani et al. [8, 10] and Williams et al. [9, 13]. The experiments concluded that the relationship between speed and concentration remained significantly identical for various network sizes. This indicated that these relationships between various network level variables were independent of network size and consistent. During their analysis they observed that the average network speed at a given concentration was lower when the intersections were operated as an un-signalized (stop-sign control) as compared to signalized intersections.

Ardekani [5] studied the two fluid characterizations urban road networks and proved the validity of these models on real urban road networks. Ardekani through field studies also concluded that the fundamental equation 3 holds true.

Recently Daganzo [14] using average network flow and accumulation suggested various recipes for improving city mobility through gridlock control. The paper proposed a relationship between the outflow (exit function (*G(n)*)) and the number of vehicles in the network. The paper derived a differential equation (equation 4) describing the number of vehicles in the network, based on the inflow (*f(t)*) and outflow (*G(n)*).

$$\frac{dn}{dt} = f(t) - G(n(t)), \quad \textbf{for } t \geq 0 \tag{4}$$

These relationships are used in the paper to determine an optimal control strategy (A-B strategy) to control inflows so as to maximize outflow. One of the practical drawbacks of this approach is that due to the stochastic nature of traffic flow there are periods where inflow is greater than the outflow, leading to eventual jam conditions, hence in the strategy proposed for efficient operations real monitoring and control of the network is required. In this paper the proposed strategy overcomes this drawback.

Geroliminis and Daganzo [15] as a continuation of Daganzo's [14] theoretical work conducted simulation experiments with the San-Francisco network. They showed a linear dependence between the travel production in the network and the outflow from the network, and an inverted U-shaped relationship between the travel production and accumulation. In addition the paper also describes the behavior of inflow with respect to accumulation. They showed that inflow remained constant till a certain degree of accumulation and then started decreasing. The paper also proposed control strategies based on real time observation of accumulation and were tested using simulation.

The network breathing strategy proposed in this paper is a cyclic process of allowing vehicles to enter the network followed by closure of their entry into the network, until the network reaches congestion. After which entrance into the network is allowed again. This process will be referred to as "network breathing". The period during which vehicles are allowed to enter is referred to as "network inhalation" and when vehicles are held from entering the network is referred to as "network relaxation". The advantage of such an approach is that the times for network inhalation and network relaxation can be predetermined depending on the network properties and would not need real time feedback.

This paper presents the derivation for the fundamental flow/speed/concentration relationship, the Greenshield's model, the Greenberg's model and the 'bell-shaped' function proposed by Drake, Schofer and May [11] at a network level. It also compares the performance of each of these models in explaining network level speed-accumulation relationships. Section 2 defines the various variables that are used in the rest of the paper. Section 3 presents the derivation for relationships between various network level variables. Section 4 outlines the methodology for the strategy. Section 5 discusses the results of applying the developed strategies to a simulated network. Section 6 concludes by discussing the advantages of these strategies and ideas for expansion of this work for further research.

DEFINITIONS

This section summarizes all the relevant variables that are used throughout this paper.

Q_i Flow on link *i*

Q_{Out} Total outflow from network

Q_{in} Total inflow into the network

K_i Concentration on link *i*

K_j Network level jam concentration

v_j Speed of the jth vehicle on the network

V_i Average speed on link *i*

V_f Inverse of the average minimum time taken to travel a mile in the network, at free flow conditions.
n_i Number of vehicles on link *i*
n Total number of vehicles on the network (sum of all n_i) (Accumulation)
n_c Number of vehicles in the network, when the outflow from the network is the maximum
n_p Maximum number of vehicles the network can accommodate at a given time
l_i Lane-mile of link *i*
l Total lane-mile in the network (sum of all l_i)
q_p The average flow in the network when number of vehicles in the network is n_p
x Number of vehicles in the network after network relaxation

The three fundamental traffic variables speed, concentration and flow at network level are defined as average over all vehicles during an observation period ε. These were defined by Ardekani [5] and Mahmassani et al. [8,10]. Average speed in a network is defined as the total number of vehicle miles traveled divided by the total number of vehicle hours in the network during an observation period ε. If there are n vehicles in the network and the velocity of the jth vehicle in the network is v_j, then the average velocity of vehicles in the network represented by *V* is:

Total vehicle-mile = $\sum_{j=1}^{n} v_j \varepsilon$

Total vehicle-hours = $n\varepsilon$

$$V = \frac{\sum_{j=1}^{n} v_j \varepsilon}{n\varepsilon} \qquad (5)$$

$$\Rightarrow V = \frac{\sum_{j=1}^{n} v_j}{n} \qquad (6)$$

The average concentration *K* in the network is defined as the total number of vehicles in the network per unit lane-mile. l is the total lane-miles of roadway in the network.

$$K = \frac{n}{l} \qquad (7)$$

The average network flow *Q* is defined as the average number of vehicles that pass through a random point in the network per unit time. The average network flow is given by:

$$Q = \frac{\sum l_i Q_i}{\sum l_i} \qquad (8)$$

In Kalfastas and Peetas work [16] on the cell transmission model they observed that on individual links the backward propagating wave speed is lower than the free flow speed, indicating that the link can never reach maximum jam density, since only a part of the available space would be filled up. Since a network is a combination of individual links, it is fair to assume that during maximum congestion, there exists a maximum accumulation (n_p) in the network and it corresponds to some minimum outflow (q_p). In Ardekani's [5] field study a maximum concentration of 30 vehicles/lane-mile was observed in the network.

With the next generation of technology of Vehicle Infrastructure Integration (V.I.I.), data of the network will be available during every time instant. Hence the average velocity, flow and density of the network would be known at every time instant. This will help provide real time state of the network, enabling us to develop real time strategies for the network.

MACROSCOPIC PROPERTIES

Measurements and relationships between macroscopic variables: flow, concentration and speed have been extensively studied for traffic streams both in theory and in field (Edie [17] and Gazis [18]). The relationships between these macroscopic variables at a network scale have been studied in simulation experiments [8, 10, 9, 13] and field studies [1]. This section provides theoretical proofs for the validity of relationships at a network scale. These relationships are then validated using microscopic simulation (VISSIM) for a network shown in Figure 1. The network is a small grid network of two lane one-way roads, the entire length of the roadway is 2.12 miles. The intersections consisted of two phase signals with a cycle length of 60 seconds. The simulation for the network for multiple runs with each run of 3240 seconds. An input of 2000 veh/hr was provided at each input. The average concentrations, speeds, flows, inflows and outflows were averaged over 120 seconds for the network. The demands were allocated to routes from origin to destinations. We also assume a fixed percentage of demand for each route on the network, to ensure that the average trip lengths are constant.

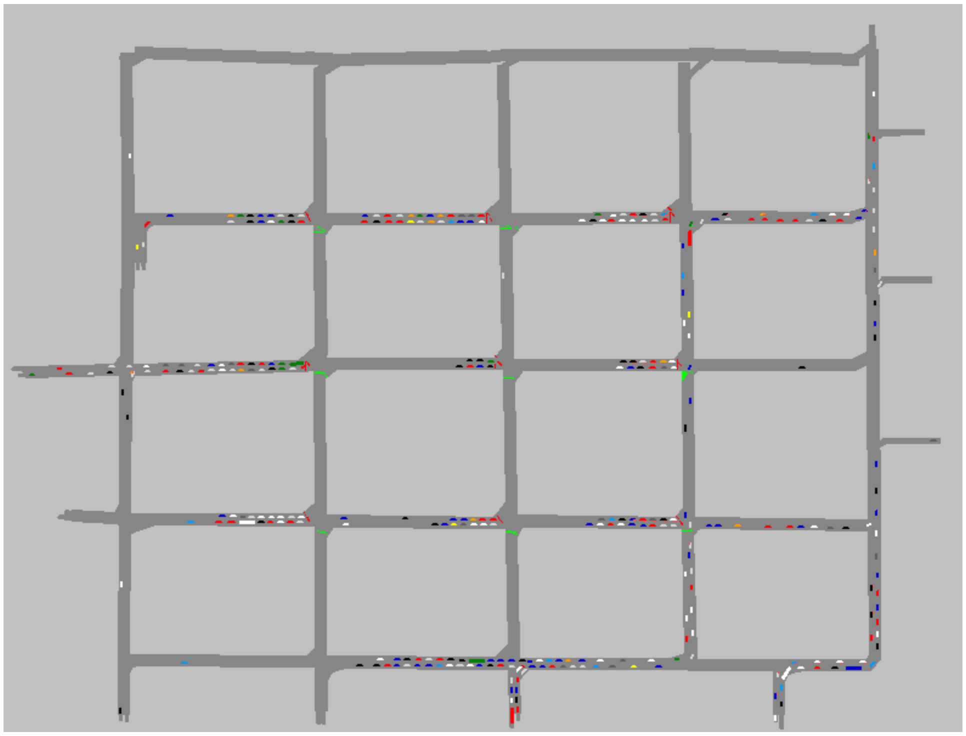

Figure 1: Simulation network considered for the study

Fundamental Network Speed/Flow/Concentration Relationship

This subsection derives the fundamental speed/flow/concentration relationship on a general network.

On an individual link i of length l_i in the network, it is known that:

$$Q_i = K_i V_i \tag{9}$$

Where V_i is the average velocity on link i defined by equation (10)

$$V_i = \frac{\sum_{\text{vehicle } j \text{ is in Link } i} v_j}{n_i} \tag{10}$$

Multiplying both sides of equation (9) with l_i, we get:

$$l_i Q_i = l_i K_i V_i \tag{11}$$

Summing both sides of equation (11) over all links i in the network and dividing by the total lane-miles in the network, we get:

$$\frac{\sum l_i Q_i}{\sum l_i} = \frac{\sum l_i K_i V_i}{\sum l_i} \tag{12}$$

It is observed that the left hand side of equation (12) is the definition for average network flow. It is also observed that $l_i K_i$ is the number of vehicles in link i. Substituting $l_i K_i$ with n_i, and the definition of average network flow in equation (12).

$$Q = \frac{\sum (l_i K_i) V_i}{\sum l_i} = \frac{\sum (n_i) V_i}{\sum l_i} \tag{13}$$

$$\Rightarrow Q = \left(\frac{\sum n_i}{\sum n_i}\right)\left(\frac{\sum (n_i)V_i}{\sum l_i}\right) = \left(\frac{\sum n_i}{\sum l_i}\right)\left(\frac{\sum (n_i V_i)}{\sum n_i}\right) \qquad (14)$$

Substitute network concentration and definition of V_i from equation (10) in equation (14).

$$Q = K\left(\frac{\sum\left(n_i\left(\frac{\sum_{\text{vehicle } j \text{ is in Link } i} v_j}{n_i}\right)\right)}{\sum n_i}\right) \qquad (15)$$

$$\Rightarrow Q = K\left(\frac{\sum \sum_{\text{vehicle } j \text{ is in Link } i} v_j}{\sum n_i}\right) \qquad (16)$$

In equation (16), $\sum \sum_{\text{vehicle } j \text{ is in Link } i} v_j$ is the sum of velocities of all vehicles on the network.

Hence by definition $\left(\frac{\sum \sum_{\text{vehicle } j \text{ is in Link } i} v_j}{\sum n_i}\right)$ is the average velocity on the network.

$$\Rightarrow Q = KV \qquad (17)$$

Equation (17) is the fundamental network speed/flow/concentration relationship.

The interesting aspect of equation 17 is that there are no inherent assumptions involved in the derivations, indicating that the fundamental network speed/flow/concentration relationship holds for a network in any state. The fundamental relationship for speed/flow/concentration should hold when the vehicles are non-homogenously loaded in the network or when the network is in a transient state.

Using data of network density and average network speed from the simulation runs the average network flow was calculated using equation 17 and was compared to the observed average network flow from the simulation. A plot (Figure 2) between the calculated and observed average network, show a perfect regression fit for $y=x$.

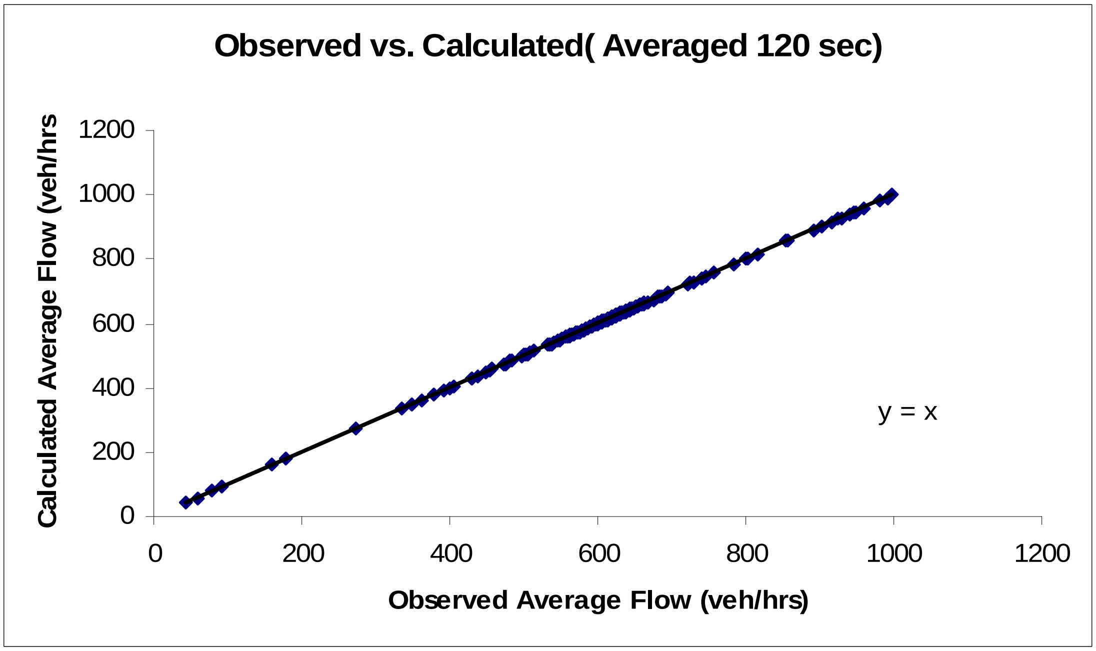

Figure 2: Observed flow vs. flow calculated using speed and density in equation 2.

Speed-Accumulation Relationship at Network level

This section derives the Greenshield's, Greenberg's and the Bell-shaped model [11] to describe the relationship between average network speed and accumulation in a homogenous network (a network in which the concentration, jam concentration and free flow speeds do not significantly differ between different links). The theoretical results are validated through simulation for the network described earlier. A plot was constructed for data points, such that vehicles are homogenously distributed.

Greenshield's Relationship

To derive the Greenshield's model for a network, it is assumed that the speed density relationship on an individual link *i* in a homogenous network follows the Greenshield's model.

$$V_i = V_f\left(1 - \frac{K_i}{K_j}\right)$$

Since the free flow speed (V_f) and jam density (K_j) do not significantly vary between links of the network.

$$\Rightarrow \sum(n_i V_i) = V_f\left(\sum n_i - \frac{\sum(n_i K_i)}{K_j}\right)$$

$$\Rightarrow \frac{\sum(n_i V_i)}{\sum n_i} = V_f\left(\frac{\sum n_i}{\sum n_i} - \frac{\sum(n_i K_i)}{K_j \sum n_i}\right)$$

Since the link densities do not significantly differ between links and is approximately equal to the network density $K_i \sim K$.

$$\Rightarrow V = V_f\left(1-\frac{\sum(n_i K)}{K_j\sum n_i}\right) = V_f\left(1-\frac{K\sum n_i}{K_j\sum n_i}\right) = V_f\left(1-\frac{(n/l)}{(n_j/l)}\right)$$

$$\Rightarrow V = V_f\left(1-\frac{n}{n_j}\right) \tag{18}$$

This proves that Greenshield's relationship holds for average network speed and accumulation in a homogenous network.

Greenberg's Relationship

To derive the Greenberg's model for a network, it is assumed that the speed density relationship on an individual link *i* in a homogenous network follows the Greenberg's model.

$$V_i = -V_f \ln\left(\frac{K_i}{K_j}\right)$$

Since the free flow speed (V_f) and jam density (K_j) do not significantly vary between links of the network.

$$\Rightarrow \sum(n_i V_i) = -\sum n_i V_f \ln\left(\frac{K_i}{K_j}\right)$$

$$\Rightarrow \frac{\sum(n_i V_i)}{\sum n_i} = \frac{-\sum n_i V_f \ln\left(\frac{K_i}{K_j}\right)}{\sum n_i}$$

Since the link densities do not significantly differ between links and is approximately equal to the network density $K_i \sim K$.

$$\Rightarrow \frac{\sum(n_i V_i)}{\sum n_i} = \frac{-\sum n_i V_f \ln\left(\frac{K_i}{K_j}\right)}{\sum n_i} = -\frac{\sum n_i}{\sum n_i} V_f \ln\left(\frac{K}{K_j}\right) = -V_f \ln\left(\frac{(n/l)}{(n_j/l)}\right)$$

$$V_i = -V_f \ln\left(\frac{n}{n_j}\right) \tag{19}$$

This proves that Greenberg's relationship holds for average network speed and accumulation in a homogenous network.

Bell-shaped Relationship

To derive the Bell-shaped model for a network, it is assumed that the speed density relationship on an individual link *i* in a homogenous network follows the Bell-shaped model.

$$V_i = V_f \exp\left[-\alpha\left(\frac{K}{K_j}\right)^d\right]$$

Since the free flow speed (V_f) and jam density (K_j) do not significantly vary between links of the network.

$$\Rightarrow \sum(n_i V_i) = \sum n_i V_f \exp\left[-\alpha\left(\frac{K_i}{K_j}\right)^d\right]$$

$$\Rightarrow \frac{\sum(n_i V_i)}{\sum n_i} = \frac{\sum n_i V_f \exp\left[-\alpha\left(\frac{K_i}{K_j}\right)^d\right]}{\sum n_i}$$

Since the link densities do not significantly differ between links and is approximately equal to the network density $K_i \sim K$.

$$\frac{\sum(n_i V_i)}{\sum n_i} = \frac{\sum n_i V_f \exp\left[-\alpha\left(\frac{K}{K_j}\right)^d\right]}{\sum n_i} = \frac{\sum n_i}{\sum n_i} V_f \exp\left[-\alpha\left(\frac{K}{K_j}\right)^d\right] = V_f \exp\left[-\alpha\left(\frac{(n/l)}{(n_j/l)}\right)^d\right]$$

$$V_i = V_f \exp\left[-\alpha\left(\frac{n}{n_j}\right)^d\right] \qquad (20)$$

This proves that relationship proposed by Drake, Shofer and May [11] holds for average network speed and accumulation in a homogenous network.

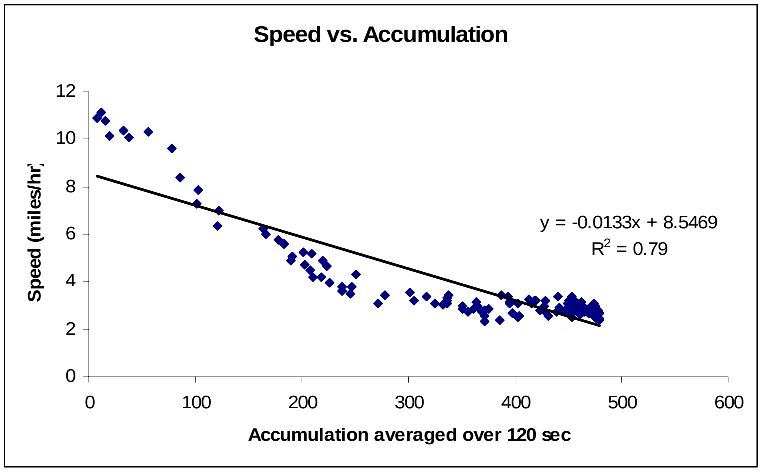

Figure 3a: Model fit for Greenshield's model for speed-accumulation relationship

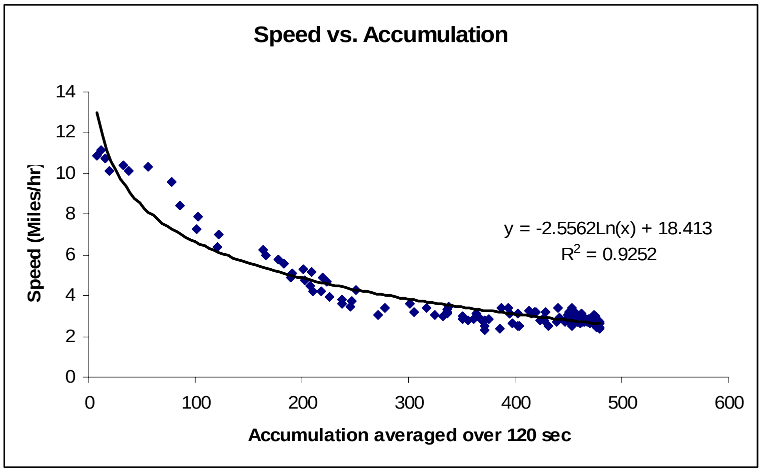

Figure 3b: Model fit for Greenberg's model for speed-accumulation relationship

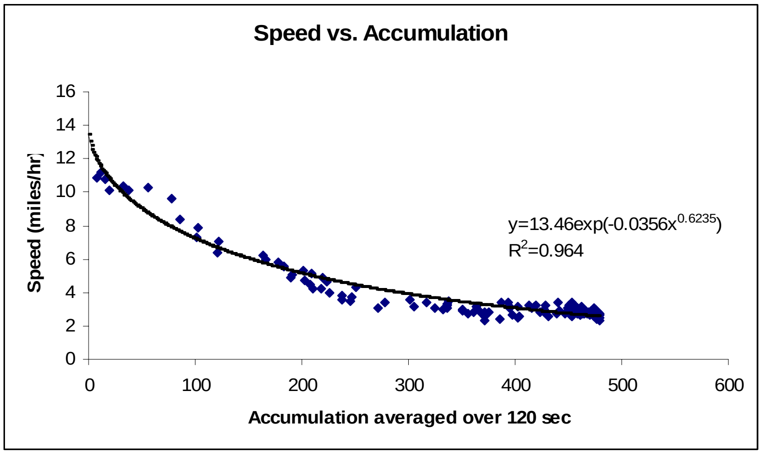

Figure 3c: Model fit for Bell-shaped model for speed-accumulation relationship

Figure 3: Model fits for various speed-density relationships

The plot (Figure 3) for the simulation results were fitted with the Greenberg's, Greenshield's and Bell-shaped model. When the results were fitted with a linear regression the R-square was found to be 0.79. A logarithmic fit showed to perform better with an R-square of 0.93. The fit for the Bell-shaped model proposed by Drake, Shofer and May [11] performed the best with an R-square of 0.964. The Bell-shaped fit for the results showed to perform significantly better in explaining the speed-accumulation relationship than Greenberg's model and Greenshield's model.

Outflow-Accumulation Relationship at Network level

To understand the relationship between outflow and accumulation, the outflow and accumulation averaged over 120 seconds collected during the simulation of the network described earlier was plotted (Figure 4).

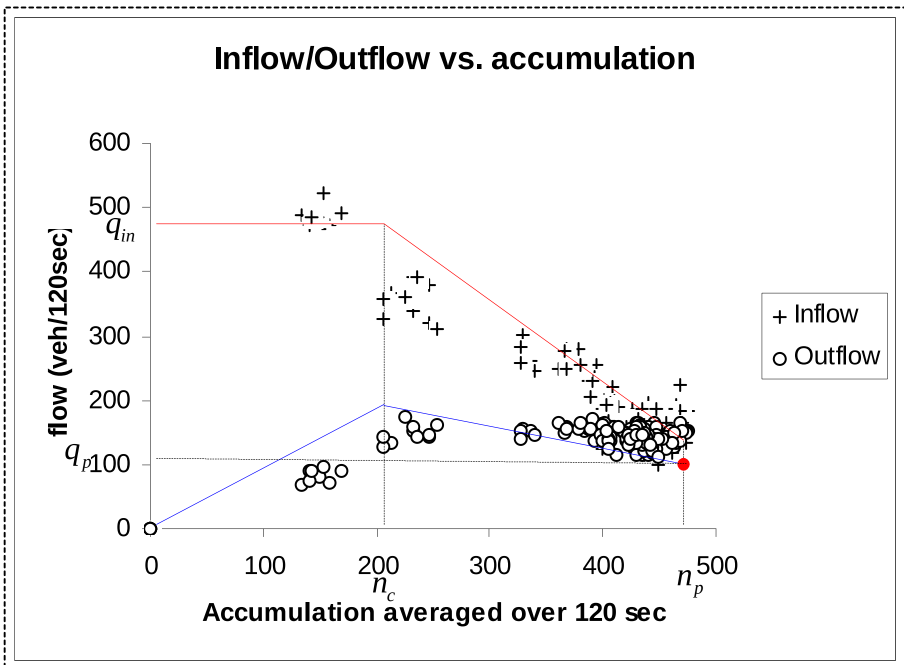

Figure 4: Plot of Inflow and outflow vs. accumulation and the trend assumed

The plot shows two distinct regimes. The first regime corresponds to the constrained regime, where the outflow increases to a maximum value (q_c=170veh/120sec) as the accumulation increases. This maximum outflow corresponds to an accumulation of n_c equal to 212veh. The second regime corresponds to the constrained regime, where the outflow decreases from its maximum value as the accumulation increases. The reason for the reduction in outflow during the constrained regime is due to blockage of exits by vehicles accessing other exits. Observing the trend in Figure 4, it is fair to assume a piecewise linear relationship to describe the two regimes of the outflow-accumulation relationship (Figure 5). The piecewise formulation describing the outflow-accumulation relationship is written as:

$$Q_{out}(n) = \begin{cases} \lambda n & \forall n \leq n_c \\ -\alpha n + \beta & \forall n \ \ s.t. \ n_p \geq n > n_c \end{cases} \tag{21}$$

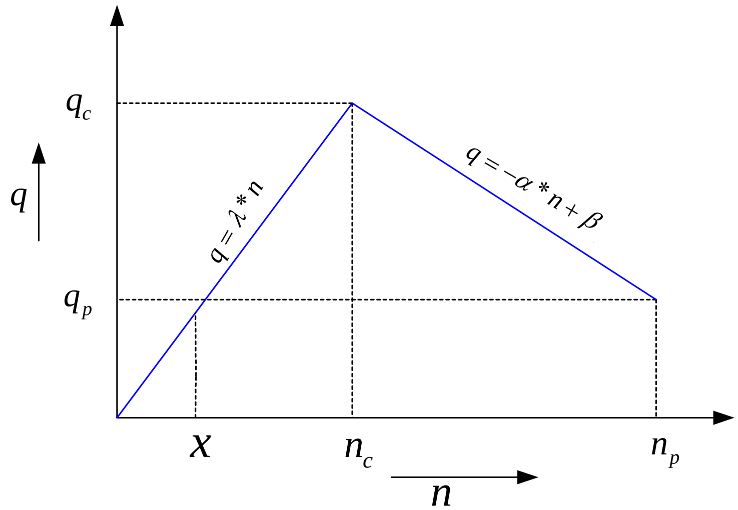

Figure 5: Conceptualization of relationship between outflow and accumulation

Inflow-Accumulation Relationship at Network level

Simulation studies by Geroliminis and Daganzo [15] showed that the dependence of inflow on accumulation had a trend similar to as seen in Figure 4. The inflow remains constant until the accumulation reaches n_c and then begins to decrease. The accumulation n_c corresponds to the boundary of the constrained and unconstrained outflow. During the unconstrained regime vehicles have an inflow equal the maximum possible flow q_{in}, since none of the entries into the network are blocked. The moment the constrained conditions begin to set in, the outflow reduces leading to queuing of vehicles and blocking of inflows, this result in the inflow being constrained. Under the constrained regime the inflow keeps reducing, as the accumulation increases until the maximum accumulation (n_p) allowed in the network. The moment the number of vehicles in the network reaches n_p, the inflow drops to the outflow q_p. Under the assumption that the network can have a maximum of n_p accumulation, if the inflow is greater than outflow (q_p) the accumulation would increase to a value greater than n_p, resulting in a violation of the assumption. Therefore the inflow will be equal to the outflow at the accumulation n_p. Therefore there is a discontinuity for the inflow at n_p. It will be later seen in this paper that if the inflow has a continuous trend and the constrained regime of the inflow intersects the outflow at the maximum accumulation (n_p), then infinite time would be taken to reach congested conditions (accumulation of n_p). In reality networks do reach congested conditions and are regularly observed in road networks around the world. To get around this problem of infinite convergence to congestion, discontinuity was assumed at the maximum accumulation (n_p). The inflow into the network is greater than q_p just before an accumulation of n_p, due to stochastic effects there are fluctuations in the accumulations, there might be periods when the total accumulation might be just lower than the maximum accumulation (n_p), therefore the traffic inflow needs to stop for brief periods, so as to maintain an average inflow of q_p. These kinds of stop and go behavior with large oscillations in flows [6] have been observed in congested conditions and can be explained by such a formulation. Therefore it is fair to assume a discontinuity. The relationship between the inflow and accumulation is shown in Figure 6, and can be formulated as:

$$Q_{in}(n) = \begin{cases} q_{in} & \forall n \;\; s.t. \;\; n \le n_c \\ -\gamma n + \sigma & \forall n \;\; s.t. \;\; n_p > n > n_c \\ q_p & n = n_p \end{cases} \tag{22}$$

A plot describing the conceptualization of the trends of the outflow-accumulation relationship and the inflow-accumulation relationship are shown in contrast to the observed values in figure 4. Since the inflow was large the network quickly progressed towards constrained condition. As can be seen in Figure 4, the vehicles remained in unconstrained regime for a brief period, during which the inflow was found to be 460 vehicles/120 sec. The inflow of vehicles just before n_p was taken to be 135veh/120 sec and the value of outflow at accumulation n_p (q_p) was found to be around 125veh/120 sec, which was also the value of the inflow at an accumulation of n_p. These values were calibrated for Do-nothing strategy.

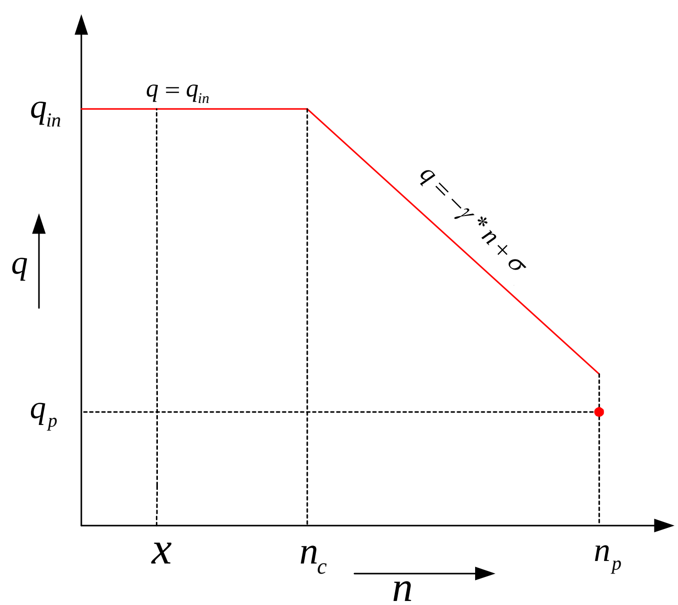

Figure 6: Plots describing relationships between inflow and accumulation

MODELING AND METHODOLOGY

This section presents a formal methodology for the network breathing strategy. In the network breathing strategy the inflow is allowed into the network until the network reaches jam conditions, after which the network is allowed to relax, implying that inflow into the network is shutdown. The intuitive reason in adopting this cyclical approach is to ensure that no sustained constrained flow (q_p) at maximum accumulation (n_p) exists. This can be observed more clearly in Figure 7 a comparison between the area under the curve between Figure 7a and 7b shows that the total number of vehicles dissipated into the network is larger with network breathing. The important aspect to determine is that relaxing the network for how long would provide the benefit being looked for.

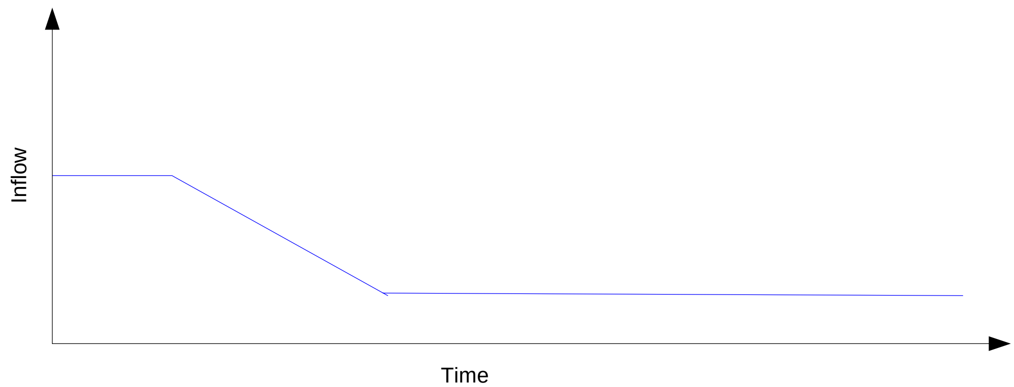

Figure 7a: Inflow profile under normal conditions

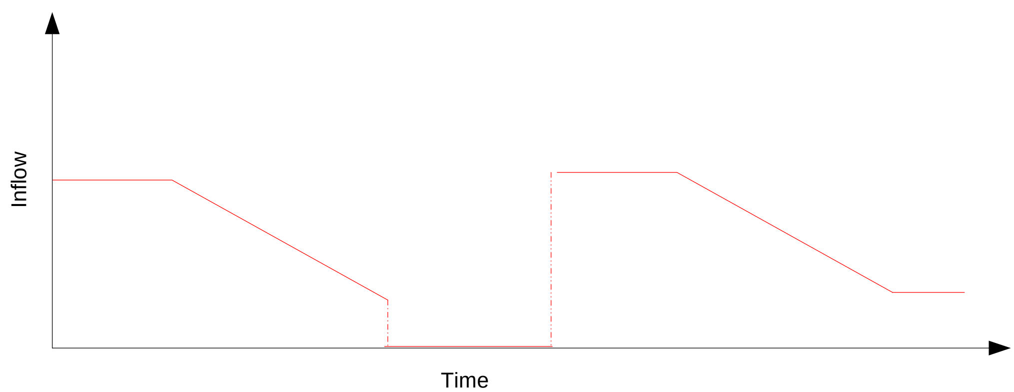

Figure 7b: Inflow profile under Network breathing scheme
Figure 7: Plots describing relationships between inflow and accumulation

The dynamics of the number of vehicles (Accumulation) in the network can be described by a differential equation (equation 23). The differential equation basically states the rate of change of accumulation is the difference between the inflow and the outflow. If the outflow is greater than the inflow then the accumulation will decrease with time, if the outflow is less than the inflow the accumulation will increase with time. It is assumed that the maximum inflow (q_{in}) is greater than the maximum outflow (q_c).

$$\frac{dn}{dt} = Q_{in}(n) - Q_{out}(n) \tag{23}$$

When vehicles initially enter the network the outflow and inflow correspond to the unconstrained regime ($n < n_c$). Therefore the time taken to dissipate vehicles during the unconstrained regime when accumulation grows from 0 to n_c, t_s is shown in equation 25.

$$\int_0^{n_c} \frac{dn}{(q_{in} - \lambda n)} = \int_0^{t_s} dt \tag{24}$$

$$t_s = -\frac{1}{\lambda}\ln\left(1 - \frac{\lambda n_c}{q_{in}}\right) \tag{25}$$

After the accumulation in the network has reached n_c, the equations describing the inflow and outflow shift to the constrained regime, where $n>n_c$. Since the maximum accumulation that can be reached in the network is n_p. The time taken for the accumulation in the network to grow from n_c to n_p is represented by t_d, in equation 26.

$$\int_0^{t_d} dt = \int_{n_c}^{n_p} \frac{dn}{((-\gamma n + \sigma) - (-\alpha n + \beta))} = \int_{n_c}^{n_p} \frac{dn}{((\alpha - \gamma)n + (\sigma - \beta))}$$

$$t_d = \frac{1}{(\alpha - \gamma)}\ln\left(\frac{(\alpha - \gamma)n_p + (\sigma - \beta)}{(\alpha - \gamma)n_c + (\sigma - \beta)}\right) \tag{26}$$

If the network has an accumulation at some intermediary value x then the time taken in the constrained regime for the accumulation to reach the maximum accumulation n_p is represented by $t_d(x)$ and is shown in equation 27. $t_d(x)$ is the time taken to reach n_p under constrained conditions, therefore the minimum value x can have during constrained conditions is n_c, but if $x \le n_c$, then the time in the constrained region is t_d in equation 26.

$$t_d(x) = \begin{cases} \dfrac{1}{(\alpha - \gamma)}\ln\left(\dfrac{(\alpha - \gamma)n_p + (\sigma - \beta)}{(\alpha - \gamma)n_c + (\sigma - \beta)}\right) & \forall x \le n_c \\ \dfrac{1}{(\alpha - \gamma)}\ln\left(\dfrac{(\alpha - \gamma)n_p + (\sigma - \beta)}{(\alpha - \gamma)x + (\sigma - \beta)}\right) & \forall x > n_c \end{cases} \tag{27}$$

It will be interesting to observe here the formulation of $t_d(x)$ in equation 27. The numerator inside the logarithmic term of equation 27, is the difference between the inflow and the outflow at the maximum accumulation (n_p), if the inflow describing the constrained region intersected the outflow at n_p without discontinuity, the value of the numerator would tend to 0, making the value of $t_d(x)$ at n_p infinite. Indicating that it would take infinite time to reach jam conditions, which is not true. For this reason it would be fair to assume a discontinuity at n_p for inflow.

According to the network breathing strategy proposed, once the network is congested with the maximum accumulation possible the network relaxation is started. During this process the inflow is shutdown, therefore the accumulation in the network begins to reduce. This increases the outflow. The basic idea behind this approach is to maintain an inflow greater than the inflow at the maximum accumulation. This helps increase the inflow into the network.

If the network is relaxed to an accumulation of x, the time taken to reach an accumulation of x from n_p depends on whether $x \le n_c$ *or* $x > n_c$ and is represented by $t_b(x)$. If $x < n_c$ then during the reduction of accumulation from n_p to n_c the outflow will correspond to the constrained regime. After which the outflow will correspond to the unconstrained regime.

Therefore the time taken to reach x if $x \le n_c$ is the sum of the time taken in the two regimes. This is shown in equation 28

$$t_b(x) = \int_{n_p}^{n_c} \frac{dn}{(0-(-\alpha n+\beta))} + \int_{n_c}^{x} \frac{dn}{(0-\lambda n)} = \int_{n_p}^{n_c} \frac{dn}{(\alpha n-\beta)} + \int_{n_c}^{x} \frac{dn}{-\lambda n} \qquad \forall x \le n_c$$

$$t_b(x) = \frac{1}{\alpha}\ln\left(\frac{\alpha n_c - \beta}{\alpha n_p - \beta}\right) - \frac{1}{\lambda}\ln\frac{x}{n_c} \qquad \forall x \le n_c \tag{28}$$

If $x > n_c$, then the drop in accumulation in the network from n_p to x occurs in the constrained regime, therefore the outflow corresponds to the constrained regime. Hence the second term in the R.H.S. of equation 28 is dropped, and since the network is relaxed to x. ($x>n_c$), n_c is replaced with x. Therefore the time taken to relax when $x > n_c$ is given by equation 29.

$$t_b(x) = \frac{1}{\alpha}\ln\left(\frac{\alpha x - \beta}{\alpha n_p - \beta}\right) \qquad \forall x > n_c \tag{29}$$

$$\therefore t_b(x) = \begin{cases} \frac{1}{\alpha}\ln\left(\frac{\alpha n_c - \beta}{\alpha n_p - \beta}\right) - \frac{1}{\lambda}\ln\frac{x}{n_c} & \forall x \le n_c \\ \frac{1}{\alpha}\ln\left(\frac{\alpha x - \beta}{\alpha n_p - \beta}\right) & \forall x > n_c \end{cases} \tag{30}$$

After the network relaxation process, the inflow is allowed back into the network, due to which the accumulation begins to increase. $t_n(x)$ is defined as the time for the network accumulation to grow from x to n_c under the unconstrained regime. This is shown in equation 31.

$$\int_{x}^{n_c} \frac{dn}{(q_{in} - \lambda n)} = \int_{0}^{t_n} dt$$

$$t_n(x) = -\frac{1}{\lambda}\ln\left(\frac{\lambda n_c - q_{in}}{\lambda x - q_{in}}\right) \qquad \forall x \le n_c \tag{31}$$

The time taken for the accumulation to grow back to n_p is the sum of the time taken for the accumulation to grow from x to n_c under unconstrained regime and the time taken for accumulation to grow from n_c to n_p under the constrained regime. If $x>n_c$, the accumulation to which network was relaxed lies in the constrained regime, since $t_n(x)$ is defined for unconstrained conditions, $t_n(x)$ is taken to be *0* for $x > n_c$. The functional form for $t_n(x)$ is given in equation 32.

$$t_n(x) = \begin{cases} -\frac{1}{\lambda}\ln\left(\frac{\lambda n_c - q_{in}}{\lambda x - q_{in}}\right) & \forall x \le n_c \\ 0 & \forall x > n_c \end{cases} \tag{32}$$

Since the inflow remains constant during the unconstrained regime the number of vehicles dissipated into the network during unconstrained conditions is the product of q_{in} and the time spent in the unconstrained region. To determine the number of vehicles dissipated into the network during the network breathing process, it is required to determine the increase in accumulation during constrained conditions.

From equation 22, the inflow during congested conditions can be described by equation 33.

$$Q_{in} = -\gamma n + \sigma \qquad \forall n > n_c \tag{33}$$

$$\frac{dn_{in}}{dt} = -\gamma n + \sigma \qquad \forall n > n_c \tag{34}$$

$$\frac{dn_{in}}{dn}\frac{dn}{dt} = -\gamma n + \sigma \qquad \forall n > n_c \tag{35}$$

$$\frac{dn_{in}}{dn} = \frac{-\gamma n + \sigma}{\left(\frac{dn}{dt}\right)} \qquad \forall n > n_c \tag{36}$$

Utilizing equation 23 and inserting the equations describing the inflows and outflows during the constrained regime. The rate of change of accumulation in the network during constrained conditions can be described by equation 37.

$$\frac{dn}{dt} = (\alpha - \gamma)n + (\sigma - \beta) \qquad \forall n > n_c \tag{37}$$

To derive a differential equation (equation 38) describing the dynamics of the number of vehicles dissipated into the network with respect to the accumulation equation 37 is replaced in equation 36.

$$\frac{dn_{in}}{dn} = \frac{-\gamma n + \sigma}{\left((\alpha - \gamma)n + (\sigma - \beta)\right)} \qquad \forall n > n_c \tag{38}$$

To determine the number of vehicles dissipated into the network from an accumulation of x to n_p ($n_{in}(x)$) under constrained conditions, is shown in equation 39.

$$n_{in}(x) = \int_x^{n_p} \frac{-\gamma n + \sigma}{\left((\alpha - \gamma)n + (\sigma - \beta)\right)} dn \quad \forall n > n_c$$

$$n_{in}(x) = \frac{-\gamma(n_p - x)}{(\alpha - \gamma)} + \frac{\gamma}{(\alpha - \gamma)}\left(\frac{\sigma}{\gamma} + \frac{(\sigma - \beta)}{(\alpha - \gamma)}\right)\ln\left(\frac{n_p + \frac{(\sigma - \beta)}{(\alpha - \gamma)}}{x + \frac{(\sigma - \beta)}{(\alpha - \gamma)}}\right) \qquad \forall x \geq n_c \tag{39}$$

When the accumulation grows from an unconstrained region, the dissipation of vehicles will be $q_{in}t_n(x)$, after which the number of vehicles dissipated under constrained conditions is $n_{in}(n_c)$. Therefore, for $x<n_c$, $n_{in}(x)$ is defined as $n_{in}(n_c)$. The functional form for $n_{in}(x)$ is given in equation 40.

$$n_{in}(x)=\begin{cases} \dfrac{-\gamma(n_p-n_c)}{(\alpha-\gamma)}+\dfrac{\gamma}{(\alpha-\gamma)}\left(\dfrac{\sigma}{\gamma}+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}\right)\ln\left(\dfrac{n_p+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}}{n_c+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}}\right) & \forall x<n_c \\ \dfrac{-\gamma(n_p-x)}{(\alpha-\gamma)}+\dfrac{\gamma}{(\alpha-\gamma)}\left(\dfrac{\sigma}{\gamma}+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}\right)\ln\left(\dfrac{n_p+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}}{x+\dfrac{(\sigma-\beta)}{(\alpha-\gamma)}}\right) & \forall x>n_c \end{cases} \qquad (40)$$

Our objective is to dissipate the maximum number of vehicles into the destination network during time *T*. The total time can then be written as the sum of the initial time taken to reach an accumulation of n_p from 0 and the product of the number of network breathing cycles taken and the time spent in each network breathing scheme. The time taken for the network breathing scheme is the sum of time required to relax the network to an accumulation of *x* ($t_b(x)$), the time required to get back to n_c ($t_n(x)+t_d(x)$). The cycles might not be able to complete the entire time period, hence during the rest of the time (*del*) the outflow is equal to the inflow (q_p). Hence *T* is written as equation 41.

$$T=t_s+t_d(n_c)+m(t_b(x)+t_d(x)+t_n(x))+del \qquad (41)$$

The number of network breathing cycles in the process of dissipation is shown in equation 42.

$$m=\left\lfloor \frac{(T-t_s-t_d(n_c))}{(t_b(x)+t_d(x)+t_n(x))} \right\rfloor \qquad (42)$$

The excess time left (*del*) after the network breathing cycles is given by equation 43.

$$del=T-\left(t_s+t_d(n_c)+\left\lfloor \frac{(T-t_s-t_d(n_c))}{(t_b(x)+t_d(x)+t_n(x))} \right\rfloor(t_b(x)+t_d(x)+t_n(x))\right) \qquad (43)$$

During this time the accumulation in the network is at the maximum and the outflow is equal to the inflow at q_p. Therefore the number of vehicles dissipated into the network during this period (*del*) is given by equation 44.

$$excessN=\begin{cases} del*q_p & del>0 \\ 0 & del\le 0 \end{cases} \qquad (44)$$

The total number of vehicles dissipated into the network during time *T*, can be written as the sum of vehicles dissipated into the network during unconstrained conditions, constrained conditions, and the number of vehicles dissipated during each cycle and during time *del*.

$$N = q_{in}t_s + n_{in}(n_c) + m(q_{in}t_n(x) + n_{in}(x)) + excessN \tag{45}$$

Replacing value of *m* in equation 45 we get equation 46.

$$N = q_{in}t_s + n_{in}(n_c) + \left\lfloor \frac{(T\text{-}t_s - t_d(n_c))}{(t_b(x) + t_d(x) + t_n(x))} \right\rfloor (q_{in}t_n(x) + n_{in}(x)) + excessN \tag{46}$$

To determine the strategy that maximizes the dissipation of vehicles into the network. *N* in equation 46 is maximized in order to determine the accumulation to which the network should be relaxed. Using this value the relaxation time and the time for network inhalation can be calculated, which can then be used to meter the inflow into the network.

RESULTS

To test the proposed above methodology the relationships between the outflows and inflows with the accumulations were calibrated for the network earlier described (Figure 1). The values observed during the analysis of the outflow vs. accumulation and inflow vs. accumulation, were used to calibrate the equation describing these models.

To determine the accumulation to which the network should be relaxed during the network breathing strategy, so that the number of vehicles dissipated into the network is maximized, equation 45 was maximized. The time unit taken was 120 seconds, since all flows and number of vehicles were averaged over 120 seconds. The maximization of equation 45 indicated that the network should be relaxed for a period of 2.25 units (2.25*120 sec) to an accumulation of 148 vehicles. A plot (Figure 8) constructed between the relaxation time and the number of vehicles dissipated into the network indicates that the dissipation of number of vehicles drops significantly quickly after the maximum. Therefore to be on the conservative side the network was relaxed for 1 unit of time (120 seconds) after which the inflow was allowed for 360 seconds.

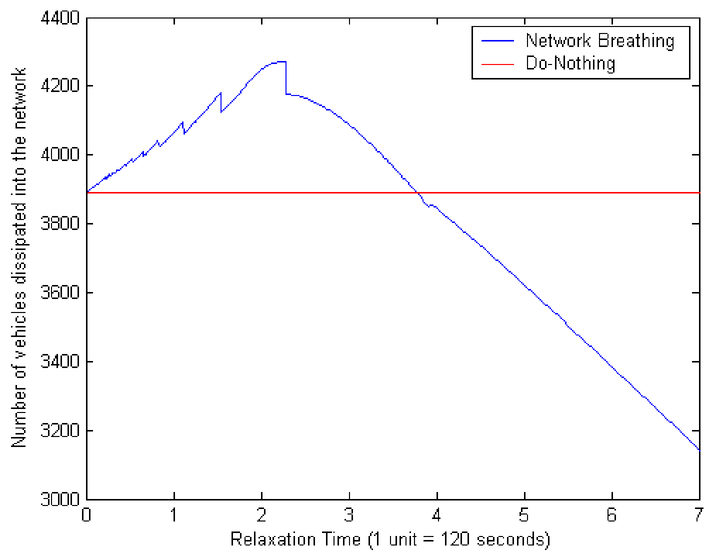

Figure 8: The plot between the number of vehicles dissipated into the network and time allowed for the network to relax.

The simulation was run for 3240 seconds for multiple runs. It was found that the average number of vehicles dissipated during the do-nothing scenario for the same time period was 3022 vehicles and for the network breathing scenario, where the network was relaxed for 120 seconds and inflow allowed for 360 seconds had a statistically significant increase in the dissipation of vehicles in the network to 4076 an increase of 154 vehicles in 3240 seconds. The simulation results showed that the network breathing strategy performs very well. Figure 9 indicates the results of the observed and predicted number of vehicles dissipated vs. the estimated. The errors printed above the observed and predicted number of vehicles in Figure 9, is less than 1%, indicating a good performance of our modeling methodology.

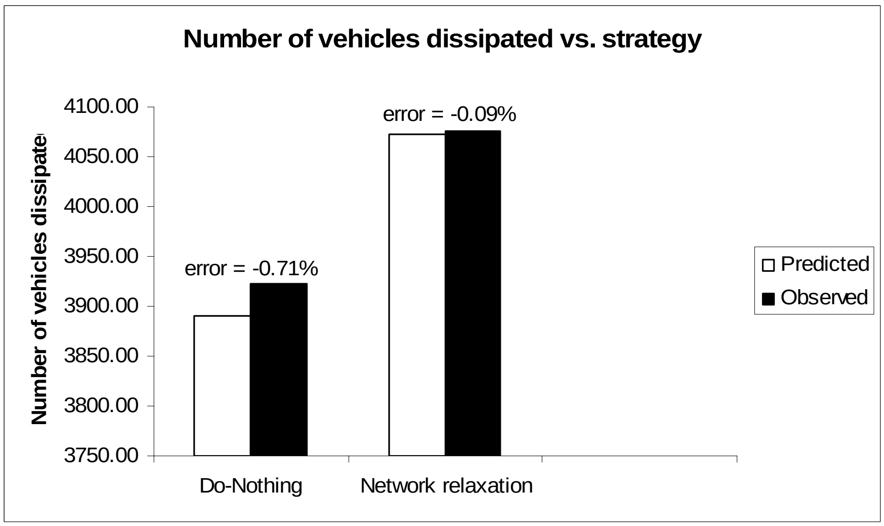

Figure 9: Results between prediction of number of vehicles dissipated into the network through theoretical methodology and actual observed number of vehicles dissipated.

CONCLUSION

This paper provides a theoretical basis to various relationships observed previously between average network level variables in simulation and field experiments. It then utilizes these relationships to develop network scale strategies that increase the dissipation of the number of vehicles into the network.

Using the theoretical framework for the network breathing strategy, the relaxation time and the network inhalation times were determined and tested using simulation. The network breathing strategy considerably outperformed a do-nothing scenario. One of the main advantages of the network breathing strategy is that it does not required real time feedback. If the properties of the network are known earlier, then the methodology described earlier can be used to come up with a prescription for relaxation time and network inhalation time. Under emergency conditions when the real time feedback might fail such an approach will be very useful. The network breathing strategy can be implemented using signals of cycle length determined by the relaxation time and the network inhalation time.

Most of the microscopic simulation studies do not consider the effect of the destination network. This results in a myopic analysis resulting in a gap between the expectations from simulation analysis and reality. The macroscopic relationships and modeling discussed need to be used in conjunction with microscopic simulation, in order for the models to incorporate effects of destination nodes.

During Evacuation it is crucial to keep the traffic moving at an acceptable rate and reduce delays. With present strategies vehicles wanting to exit onto a particular destination, are constrained by the state of the destination network. This results in backups which result in reduction of flows on the mainstream evacuation route, hindering flow of vehicles wanting to go to a destination further downstream on the evacuation route. The network breathing scheme can be used to effectively increase the dissipation into the destination networks. During the closure of the inflow into one destination network the vehicles can be redirected to another destination, hence improving the flow on the evacuation routes.

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