

Calibrating Speed-Density Functions for Mesoscopic Traffic Simulation

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Abstract

Mesoscopic traffic simulation models combine macroscopic supply (e.g. link performance functions and capacities) with microscopic demand (e.g. individual drivers and disaggregate behavior models) to capture the time-varying evolution of congestion patterns, queues and spillbacks on traffic networks. Such systems are being applied to solve a variety of off-line and on-line traffic management problems. Often, the link performance function takes the form of a speed-density relationship for each link or segment in the study network. These functions are based on the fundamental diagram and model the variation of average vehicle speed with traffic density. Since each network link or segment can have a separate speed-density function (each with a few parameters), the total number of parameters to be calibrated is potentially very large. This paper presents some experiences with calibrating speed-density functions on a variety of real networks. Flexible off-line and on-line calibration frameworks are discussed. The calibrated functions are embedded within the DynaMIT dynamic traffic assignment model, whose accuracy is evaluated against real data. The expected benefits from re-calibrating speed-density functions during on-line operations using real-time sensor data are also illustrated.

Introduction

Mesoscopic simulation tools such as DynaMIT (Ben-Akiva et al., 2002) and DYNASMART (Mahmassani, 2001) have been developed to model large-scale traffic networks. Such dynamic traffic assignment (DTA) tools combine detailed demand and supply model components together with their complex interactions to provide accurate depictions of queues, spillbacks and congestion evolution. Macroscopic simulation models also are used for modeling freeway corridors (e.g. Messmer and Papageorgiou, 2001; Ngoduy and Hoogendoorn, 2003). The demand components of these models typically include time-varying origin-destination (OD) matrices and route choice models, while the supply side captures link capacities, queuing models and macroscopic traffic dynamics relationships. Critical for modeling traffic dynamics are speed-density functions that play an important role in both mesoscopic and macroscopic traffic simulation models.

Numerous studies have focused on estimating various demand-side inputs and parameters. For example, many papers focus on the estimation of dynamic OD flows from traffic counts (Okutani, 1987, Cascetta et al., 1993, Ashok and Ben-Akiva, 1993, 2000, 2002, Balakrishna and Koutsopoulos, 2008). Route choice models have been estimated from surveys (e.g. Ramming, 2001) and refined with aggregate data (Balakrishna et al., 2005; Tsavachidis, 2000). The supply side seems to have received relatively less attention, though it plays a critical role in determining network performance. Speed-density functions are particularly challenging, as they must encapsulate a variety of effects including traffic dynamics, lane speed distributions, vehicle and driver mix, and weather conditions. Many of these aspects vary with location within the network, and require careful calibration against real-world sensor data.

This paper reviews methods for calibration of speed-density relationships and presents some experiences with calibrating speed-density functions in the context of Dynamic Traffic Assignment (DTA) models, on a variety of real networks including Irvine and Los Angeles (California), Lower Westchester County (New York) and Southampton (UK). Both off-line and on-line calibration frameworks are discussed, that allow the incorporation of any available source of data, including data from Automated Vehicle Identification (AVI) systems and probe vehicles. The calibrated functions are embedded within the DynaMIT dynamic traffic assignment model, whose accuracy is evaluated against real data. The expected benefits from re-calibrating speed-density functions during on-line operations using real-time sensor data are illustrated.

Approaches for calibration of speed-density relationships

The calibration of a DTA's speed-density functions involves a potentially large set of parameters. Recent studies have employed systematic algorithms for the calibration of DTA supply models, in particular speed-density relationships, with varying degrees of success. The typical data used for the calibration of these parameters are sensor records of at least two of the three primary traffic descriptors: speeds, flows (or counts) and densities (or detector occupancies). In this section we review the experience with optimization algorithms applied to this problem. We classify the applications as off-line (archived sensor data) and on-line (real-time sensor data and calibration).

Off-line calibration approaches

Link performance functions in much of the literature are calibrated by fitting a curve to the observed traffic data. For example, Leclercq (2005) estimates four parameters of a two-regime flow-density function with data from arterial segments in Toulouse, France. The function is comprised of a parabolic free-flow part and a linear congested regime. An interior point, conjugate gradient method is employed to optimize the fit to observed sensor flows, with the fitted flows obtained from the assumed flow-density function. Van Aerde and Rakha (1995) describe the calibration of speed-flow profiles by fitting data from loop detectors on I-4 near

Orlando, Florida. Links without sensors are allotted a speed-flow profile from a physically similar link that is instrumented (and for which a profile was fitted).

A major drawback of the above approach is one of localized fit. The estimated link performance functions reflect spot measurements at discrete sensor stations, and do not necessarily correspond to overall link dynamics (especially in the presence of congestion). The estimation procedure also does not enforce consistency across contiguous links or segments, stressing the need for an expanded approach that considers larger sections of the network. For example, it may be beneficial to relax the fit at a lightly traveled link so as to improve the fit at several links further downstream.

In the context of traffic simulation and DTA models, most calibration approaches focus on the independent estimation of subsets of supply parameters. Munoz et al. (2004) describe a calibration methodology for a modified cell transmission model (MCTM), applied to a 14-mile westbound stretch of the I-210 freeway in Pasadena, CA. Free-flow speeds are obtained through least squares, by fitting a speed-flow plot through each detector's data. Free flow speeds for cells without detectors are computed by interpolating between the available speed estimates. In the case of bad or missing sensor data, a default of 60 mph was assumed. Speed-flow functions are obtained through constrained least squares on sensor data from congested cells.

Many applications of macroscopic traffic models focus on freeway corridors or sections. Ngoduy and Hoogendoorn (2003) calibrate ten METANET parameters for a section of the A1 freeway in The Netherlands using the Nelder-Mead method (a gradient-free algorithm working directly with objective function evaluations). The calibrated terms include fundamental diagram parameters such as free-flow speed, minimum speeds and maximum density, and other coefficients that capture the effects of merging, weaving and lane drops. Ngoduy et al. (2006) calibrate six of these METANET parameters for a freeway section with no ramps. An objective function measuring the fit to count and speed data is minimized.

Park et al. (2006) apply DynaMIT to a network in Hampton Roads, VA and estimate speed-density functions for segments. They adopt the procedure in Van Aerde and Rakha (1995) and conclude that the initial calibration results need adjustments to improve DynaMIT's overall ability to estimate and predict traffic conditions.

Kunde (2002) describes a three-stage approach to speed-density calibration. At the disaggregate level, segment speed-density relationships are estimated similarly to Van Aerde and Rakha (1995). In the second stage, a suitable sub-network is chosen, and the estimates from the previous stage are refined by accounting for interactions between the segments. The choice of a subnetwork depends on the structure of the network and the location of sensors. An ideal sub-network would allow one to deduce the true OD flows for the sub-network from the available

sensor count information, so that the supply parameters may be inferred under known demand conditions. The final stage utilizes the entire network to incorporate demand-supply interactions into the calibration process. The approach was demonstrated with the DynaMIT DTA model, using data from Irvine, CA. A total of 1373 segments were divided into 11 groups and a speed-density function fitted for each group.

A least squares objective function measuring the fit to count data was then minimized on a sub-network with known demands inferred from sensor counts. SPSA (Simultaneous Perturbation Stochastic Approximation) was used to fine-tune the speed-density function parameters. This algorithm approximates the components of the gradient vector from just two objective function evaluations, after perturbing all components of the parameter vector simultaneously (Spall, 1994). The Box-Complex algorithm (Box, 1965) was applied with better success, though the numerical example was too small to draw general conclusions.

In the previous approaches, the calibration variables were limited to the speed-density function parameters. However, the optimization depends on several other DTA inputs such as OD flows and route choice model parameters. The values selected for these other inputs and parameters thus impact the outcome of the supply calibration. One may thus iterate between demand and supply calibration steps until convergence (as defined by the modeler) is reached. This iterative procedure can be time-consuming and inefficient, as only a subset of the available data is used in either calibration.

Balakrishna (2006) and Balakrishna et al. (2007) present a calibration framework that allows the simultaneous calibration of all supply and demand parameters and unknown inputs typical to DTA models (e.g. OD flows, route choice parameters, capacities, speed-density parameters) using any available data (e.g. counts, speeds, densities, queue lengths). Thus all significant DTA inputs and parameters may be estimated simultaneously, providing the most efficient result. The problem is solved with the SPSA algorithm.

This calibration approach provides a unique advantage. Since the parameters of the speed-density functions for all segments are estimated simultaneously, the function parameters for each segment can be calibrated to better fit the traffic data at the network level.

On-line calibration approaches

In the current DTA framework, only the OD flows are calibrated on-line. In most cases, the approach to the problem of calibration of the other parameters has been to calibrate the simulation models off-line using a database of historic information. The calibrated parameter values are then used in the on-line simulations. The calibrated model parameters, therefore, represent average conditions over the period represented in the data. Models that were calibrated this way may produce satisfactory results in off-line evaluation studies, which are concerned with the expected performance of various traffic management strategies. However, this may not

be the case in real-time applications, which are concerned with the system performance on the given day. If the model that was calibrated off-line is used without adjustment, the system is not sensitive to the variability of the traffic conditions between days, which are the result of variations in the parameters of the system, such as weather and surface conditions. Such variations may cause traffic conditions to differ significantly from the average values. Thus, the predictive power of the simulation model may be reduced. To overcome this problem, real-time data can be used to recalibrate and adjust the model parameters on-line so that prevailing traffic conditions can be captured more accurately. The wealth of information included in the off-line values can be incorporated into this process by using them as a priori estimates.

Doan *et al.* (1999) outline a framework for periodic adjustments to a traffic management simulation model to maintain an internal representation of the traffic network that is consistent with that of the actual network. A similar approach is proposed in Peeta and Bulusu (1999), where consistency is sought in terms of minimizing the deviations of the predicted time-dependent path flows from the corresponding actual flows. He *et al.* (1999) develop an on-line calibration process that complements their off-line approach and adjusts an analytical dynamic traffic model's output to be consistent with real-world traffic conditions.

Tavana and Mahmassani (2000) use transfer function methods (bivariate time-series models) to estimate dynamic speed–density relations from typical detector data. Huynh *et al.* (2002) extend the work of Tavana and Mahmassani (2000) by incorporating the transfer function model into a simulation-based DTA framework. Qin and Mahmassani (2004) evaluate the same model with actual sensor data from several links of the Irvine, CA network.

Wang and Papageorgiou (2005) present a general approach to the real-time estimation of the complete traffic state in freeway stretches. They use a stochastic macroscopic traffic flow model and formulate it as a state-space model, which they solve using an EKF. The formulation allows dynamic tracking of time-varying model parameters by including them as state variables to be estimated. A random walk is used as the transition equations for the model parameters. Wang *et al.* (2007) present an extended application of this approach.

Antoniou *et al.* (2007) formulate the problem of on-line calibration of a DTA model as a non-linear state-space model that allows for the simultaneous calibration of all model parameters and inputs. The methodology is generic and flexible and does not make any assumptions on the underlying model structure, the parameters to be calibrated or the type of available measurements. Because of its nonlinear nature, the resulting model cannot be solved by the Kalman filter, and therefore, nonlinear extensions are considered: the extended Kalman filter (EKF); the limiting EKF (LimEKF); and the unscented Kalman filter. The solution algorithms are applied to the on-line calibration of the state-of-the-art DynaMIT DTA model, and their use is demonstrated in a freeway network in Southampton, U.K. The LimEKF shows accuracy that is comparable to that of the best algorithm but with vastly superior computational performance.

Case Studies and results

We now present results related to the application of the state of the art methods reported in Balakrishna et al. (2007) and Antoniou et al. (2007). These studies respectively focus on the off-line and on-line calibration of speed-density parameters used in the DTA model DynaMIT. The focus of the discussion is on the impact of data used in the calibration and the importance of calibration with respect to the ability of the model to replicate actual conditions. In all studies the quality of the calibration was ascertained using the Normalized Root Mean Square Error (RMSN) statistic to evaluate the fit to both count and speed data:

$$RMSN = \frac{\sqrt{S \sum_{i=1}^S (y_i - \hat{y}_i)^2}}{\sum_{i=1}^S y_i} \quad (2)$$

where y_i are observed measurements, \hat{y}_i are simulated values and S is the total number of measurements.

Off-line calibration results

The above method has been applied to the Los Angeles network (Figure 1), an area of heavy traffic throughout the year owing to commuters and the regularity of sporting and convention special events. The network has 740 segments.

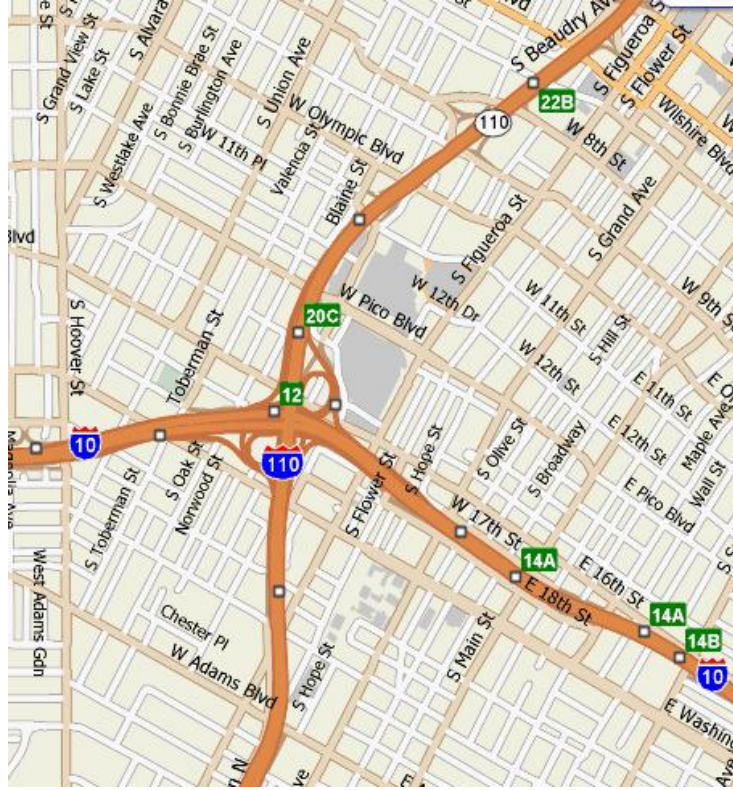


Figure 1: Los Angeles Network [source: ww.mapquest.com]

Table 1 summarizes the numerical results. $RMSN^c$ represents the fit to counts while $RMSN^s$ is the fit to speeds. $RMSN$ was used to evaluate the quality of the results. It is seen that the estimator $S(c)$, denoting supply calibration using count data, results in a significant improvement in replicating the counts and traffic dynamics (speeds) in the area. Further, the increased accuracy is reflected on both freeway and arterial links. In the base case (Ref), the speed-density parameters were fitted at individual sensor locations and attributed to all segments in the respective groups.

Estimator	Fit to Counts ($RMSN^c$)		Fit to Speeds ($RMSN^s$)	
	Freeway	Arterial	Freeway	Arterial
Ref	0.218	0.239	0.181	0.203
S(c)	0.149	0.178	0.119	0.131

Table 1: RMSN statistics for Los Angeles study

Vaze (2007) presented a case study, using the framework suggested by Balakrishna et al. (2007), in which demand and supply parameters were simultaneous calibrated using not only loop detectors data but also AVI data. In the network of Lower Westchester County, NY (shown in Figure 2), sensors deployed for an electronic toll collection system at various locations detect vehicles equipped with transponders and report, among other information, point-to-point travel time for specific locations. Using synthetic data for

this setup, Vaze calibrated a total of more than 6400 parameters (3856 demand parameters, i.e. 482 OD pair for each of the 8 intervals of simulation, and 2624 supply parameters, including 10 groups of parameters for speed-density relationship). SPSA was used as the solution algorithm.

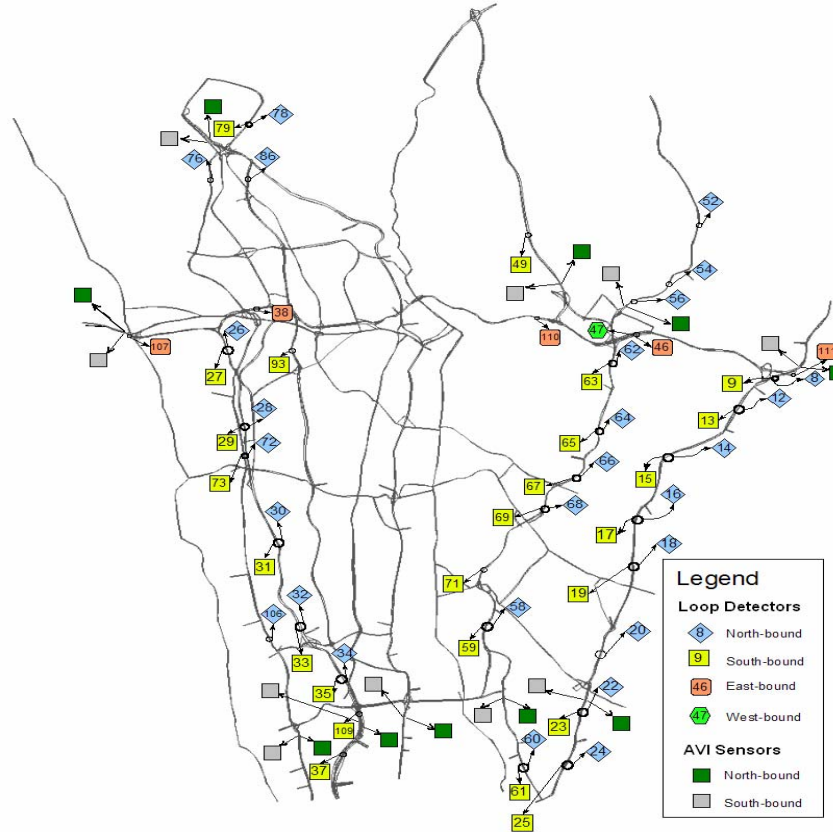


Figure 2: Lower Westchester County Network

Vaze confirmed that simultaneous demand-supply calibration was found to be superior compared to the demand-only calibration and it increased the calibration accuracy substantially. He also found that the use of AVI data had improved the calibration accuracy, both in terms of sensor count error as well as travel time (Table 2).

	Fit to counts (RMSN ^c)	Fit to speed (RMSN ^s)
A priori	0.253	0.291
Count data	0.200	0.222
Count+AVI data	0.182	0.212

Table 2: RMSN statistics for Lower Westchester County study

On-line calibration results

Antoniou et al. (2007) present an application of their on-line DTA calibration methodology using data from a freeway network in Southampton, UK, (Figure 3). The study demonstrates the

performance gains that can be obtained through the dynamic, simultaneous calibration of the speed-density relationships and other supply-side parameters.

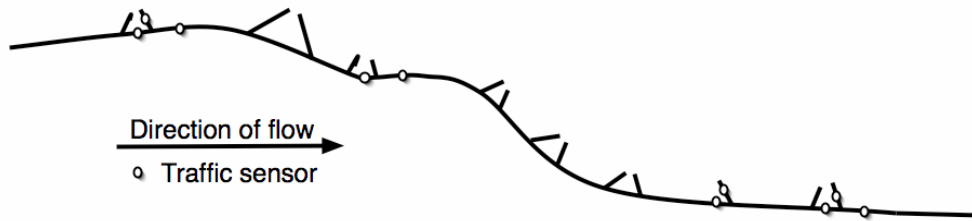


Figure 3: Lower Westchester County Network

Table 3 summarizes the DTA model estimation accuracy improvements due to the impact of the on-line calibration of the supply parameters. The base case in this table is the situation where only the demand parameters (OD flows) are calibrated on-line. The initial off-line calibration of the DTA model has been based on data from several days with normal (dry) weather conditions. When the on-line calibration is performed for a day with similar environmental conditions, the simultaneous on-line calibration of both the demand and supply parameters results in an improvement of the model estimation accuracy of more than 10% both in terms of fit to counts and fit to speed (over the base case in which only the demand parameters are calibrated on-line).

The power of the on-line calibration procedure is further demonstrated by applying the same model (initially calibrated using data from days with dry weather) to a rainy day. Again, the on-line calibration allows the model to adapt and capture the prevailing conditions satisfactorily.

	Demand-only		Demand and supply	
	Fit to counts (RMSN ^c)	Fit to speeds (RMSN ^s)	Fit to counts (RMSN ^c)	Fit to speeds (RMSN ^s)
Dry weather	0.128	0.126	0.109	0.112
Wet weather	0.115	0.131	0.102	0.117

Table 3: RMSN statistics for Southampton study

Figure 4 presents examples of the estimated speed-density relationships. As expected, the “average” curve falls between the relationships obtained for intervals with dry and wet weather conditions respectively. For the same density, lower speeds are experienced under wet weather conditions, while in general dry weather conditions allow for higher speeds for the same density values.

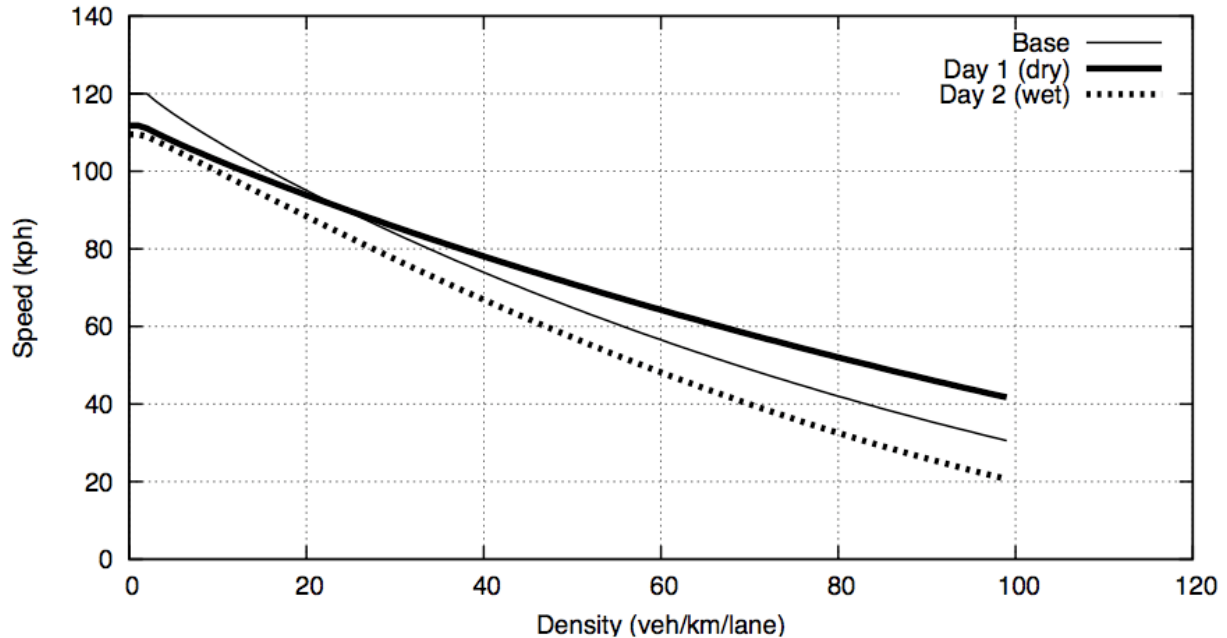


Figure 4: Speed-density relationships for average, dry and wet weather conditions

Conclusion

Speed-density functions are important supply-side inputs to many DTA models, and help capture traffic dynamics. Owing to the wide variation in the factors that affect traffic dynamics (such as merging and weaving behavior, vehicle and driver mix, weather conditions, etc), these functions must be calibrated with traffic data. This paper reviews some applications of the DynaMIT DTA model in a variety of locations, focusing on the calibration of its segment-specific speed-density functions.

The local calibration of speed-density functions is generally straightforward, but has some potential drawbacks. The functions can be over-fit in each specific location resulting in a sub-optimal calibration at the network level. Grouping of segments due to sparse sensor coverage can also result in the discarding of spatial differences. Further, the availability of modern data such as point-to-point travel times can help to better explain traffic dynamics. A flexible calibration approach that addresses these aspects is reviewed, and its benefits outlined. Case studies drawn from real-world applications of DynaMIT, a mesoscopic DTA model, are presented to illustrate that speed-density functions can be accurately and reliably calibrated with a wide range of data. The validity of these calibrations is confirmed through off-line and on-line tests of state estimation and prediction performance.

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