

Fundamental diagram on urban roads – myth or truth?

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Abstract

There is an ongoing discussion about the suitability of fundamental diagrams[1] in urban networks. We have argued in favor of such a fundamental diagram [2], while some researchers deny that there is such a thing like a fundamental diagram on urban roads, since for any level of demand any travel time is possible [3]. This contribution tries to sort this out by referring to new data sources as well as to simulation approaches. We hypothesize that in fact there is a fundamental diagram in urban roads, however, there is no one-to-one correspondence to the fundamental diagram on freeways.

1 Introduction

The familiar fundamental diagram on freeways [1], in its most basic form, relates the traffic flow q to the traffic density k . In practice, the density is not measured directly (even double loop detectors can't do it), so most often the plot of the (averaged) traffic speed v against traffic flow is used as fundamental diagram. Regarded as a simple plot of the two values against each other, the fundamental diagram is quite useful. It is another story whether the averaged fundamental diagram, which of course yields a simple one-dimensional curve $q(k)$ does have any physical or statistical meaning. In the sense of Lighthill Whitham theory [4] this curve is more than just a statistical average, it is a kind of limiting curve which is found if only equilibrium situations of traffic are recorded and non-equilibrium is excluded. This is basically at the core of the discussion brought up in [5].

Traffic flow in a city might be different, since traffic flow is now strongly dominated by bottlenecks, which may completely override any pure dynamical traffic flow effect: usually, jam on a freeway have spatial extensions way bigger than the typical distance between intersections in a city. If a city is regarded as a complicated system of links and nodes, one may regard a fundamental diagram as a kind of input output relationship, with travel time (or the related delay d , or travel speed V) as measure of efficiency. Simplifying this to a basically one long link (think of an arterial) with impurities in between (the traffic lights, intersections, and the like), then what's going on in this system depends on the inflow q_{in} , the outflow q_{out} , and of course the nasty details within which may be described as a fairly complicated travel time function $tt()$. Interestingly, density has vanished from this description, in difference to the freeway case density depend on the boundaries q_{in} , q_{out} , but also on the details of the intersection design.

Note, that we may define equilibrium in a very broad sense here: it is the case where $q_{\text{in}} \approx q_{\text{out}}$ holds, and all other internal things remain constant.

Note also, that for the case of a single intersection, there is already a very good theoretical description available, although it is usually not regarded as such: the delay time d , and with it the travel time $tt()$ can be quite accurately being described by Webster's equation [6], which relates demand q_{in} to travel time, and, along with this, to travel speed.

$$d(q := q_{\text{in}}) = \frac{C}{2} \frac{\left(1 - \frac{G}{C}\right)^2}{1 - \frac{C}{G} \frac{q}{Q}} + \frac{\left(\frac{q}{Q}\right)^2}{2q \left(1 - \frac{q}{Q}\right)} - 0.65 \left(\frac{C}{q^2}\right)^{\frac{1}{3}} \left(\frac{G}{C}\right)^{2+5\frac{G}{C}} \quad (1)$$

where Q is the maximum flow (throughput), G is the effective green time, and C is the cycle time of the signal.

In this case, a kind of fundamental diagram can be defined quite clearly, however, in this case it is only the mean value which is simple: the distribution of delay $p(d)$ does not follow any simple Gaussian distribution as is the case with the freeway fundamental diagram. This is due to the fact, that the distribution is clearly bi-modal: a vehicle has a certain change to pass the intersection without stopping (peak around small delays) or it can be stopped by the intersection, yielding a large delay.

The aforementioned fundamental diagram can be now defined by noting that the travel time and the travel speed V are related to e.g. Webster's delay approximately (where v_{free} is the speed limit on that link):

$$V(q) = \frac{L}{L/v_{\text{free}} + d(q)}. \quad (2)$$

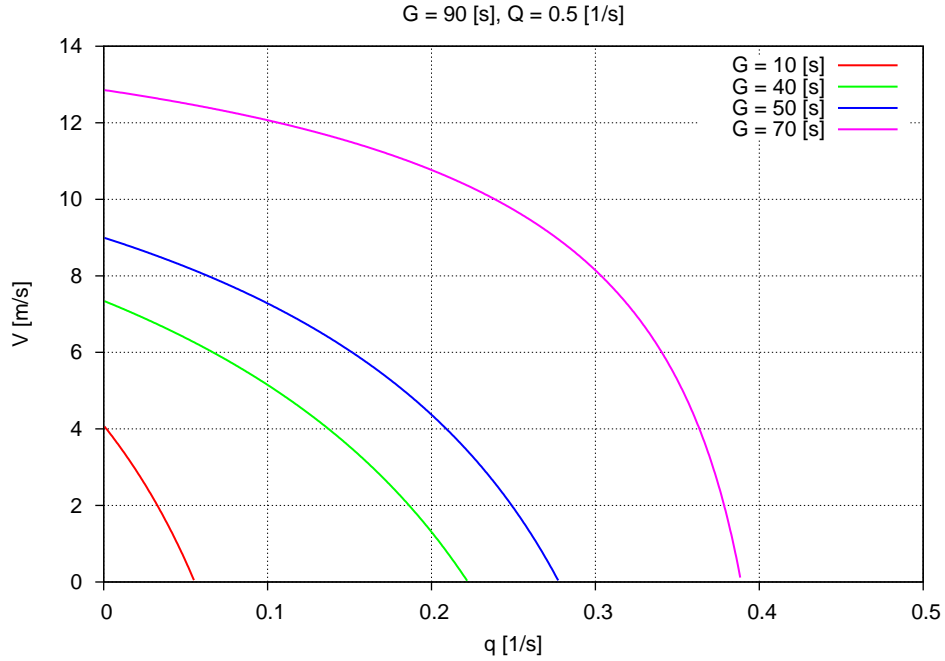


Figure 1: Travel speed versus demand for a fixed cycle signal and for various green times. For the plot, only the first two terms in Webster's equation have been used, since the third term causes strange behaviour for small q .

which is depicted in Fig. 1.

(This expression is an approximation only, since in reality a very complicated pattern of stopped and free flowing vehicles results on an over-saturated link.)

Note, that two assumptions have been made here, which do not hold in general:

- The Eq. (1) assumes that $d(q)$ goes to infinity for $q \rightarrow \frac{G}{C}Q$; since there is no infinite travel time in reality, also the travel speed never goes to zero. If one defines travel time as the time a vehicle needs between entering and leaving a link, then the maximum possible travel time is of course bounded by N_{\max}/q_{out} , where $q_{\text{out}} \leq \frac{G}{C}Q$
- this is only true, if nothing ugly happens down-stream, which in essence lowers Q . In any case, when a link is over-saturated, the up-stream and downstream links must be taken into consideration, too.

2 Data sources used

There are a couple of data sources available that can be used to address the topic of this work. Here, the aggregated data recorded in 2003 along a Cologne arterial ("Aachener Straße") will be used. Unfortunately, they cannot be directly compared to the simple Webster-based fundamental diagram above, because between the points for recording the vehicles were several signals. (The distances between the measuring points were between 300 m and 1,900 m.) Therefore, we have to resort to simulation results later on. The data have been recorded for ten days in March 2003, and along with the travel times, the traffic flows have been measured. While travel times have been aggregated into 30 s bins, traffic flows have been aggregated into 5 min bins. Fig. 2 shows two links with different maximum speeds.

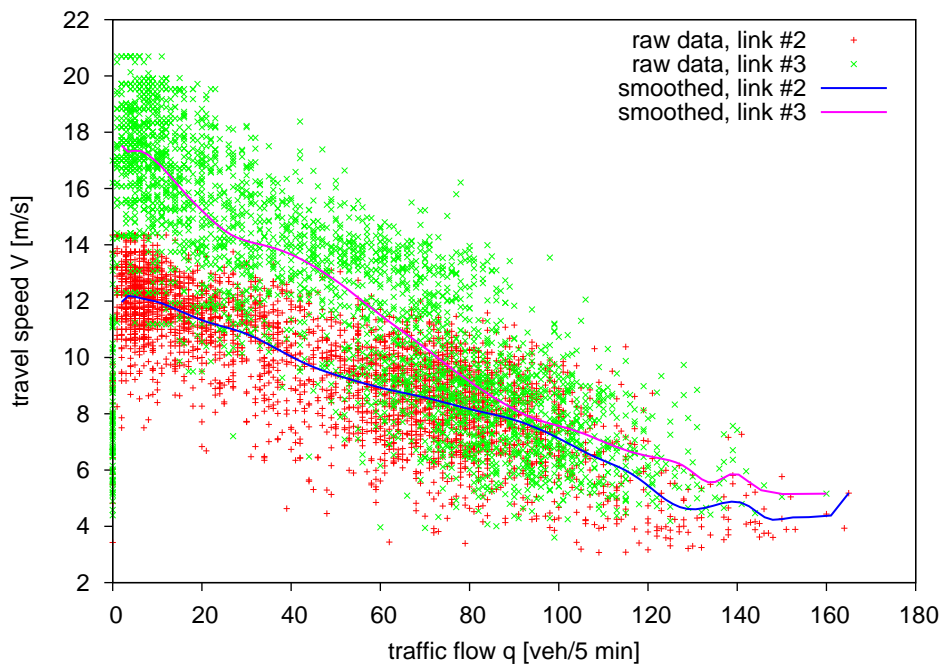


Figure 2: Travel speed versus demand for a real arterial in Cologne. See text for details.

References

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