A Stochastic Macroscopic Modeling Framework to interpret the fundamental diagram

Serge P. Hoogendoorn, PhD; Hans van Lint, PhD; Victor Knoop, Msc

Abstract:
Modeling breakdown probabilities or phase transition probabilities is an important issue when assessing and predicting the reliability of traffic flow operations. Looking at empirical spatio-temporal patterns, these probabilities clearly are not only a function of the local prevailing traffic conditions (density, speed), but also of time and space. For instance, the probability that start-stop wave occurs generally increases when moving upstream away from the bottleneck location.

This contribution proposes a unifying stochastic modeling framework that allows us to model the dynamics of the breakdown or phase-transition probabilities in an intuitive manner using the kinematic wave model (Lighthill and Whitham,1955) as a basis. Different researchers have considered the dynamic modeling of breakdown probabilities (see (Heideman,2002), (Brilon et al,2005), (Kühne and Manke,2005), (Kerner,2006)), commonly using (stochastic) queuing analysis and nucleation models; we refer to (Kerner,2006) for a thorough discussion of these modeling approaches. In doing so, we propose using *coupled set of partial differential equations* describing the traffic dynamics and the dynamics of the phase-transition probabilities p(t,x). We argue that the proposed modeling framework can be considered as a straightforward generalization of the kinematic wave theory to three phase theory.

The main result is that we can reproduce the main characteristics of the breakdown probabilities and related traffic flow operations. This is illustrated by means of two examples: free flow to synchronized flow (F-S transition) and synchronized to jam (S-J transition). We show that the probability of an F-S transition increases away from the on-ramp in the direction of the flow; the probability of an S-J transition increases as we move upstream in the synchronized flow area.

References

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