Traffic Flow Theory Historical Research Perspectives

S.L. DHINGRA[1] & ISHTIYAQ GULL [2]

1.Prof. (Retd.) Transportation Systems Engg & Emeritus Fellow, IIT Bombay-400076
Email ID: sl.dhingra@gmail.com

2. M.Tech. Student, Transportation Systems Engg. IIT Bombay-400076
Email ID: ishtiyaqgull@gmail.com

Submitted to: Greenshields Symposium, Woods Hole, MA- July2008

Date of Submission= 4th June, 2008
Word Count=5971 words + 5 figures+ 1table=7471words

ABSTRACT

Traffic flow is a kind of many-body system of strongly interacting vehicles. Traffic jams are a typical signature of the complex behavior of vehicular traffic. Various mathematical models are presented to understand the rich variety of physical phenomena exhibited by traffic. This paper provides an overview of what is currently the state -of- the-art with respect to traffic flow theory. Starting with a brief history about vehicular traffic flows, discuss the Greenshields, Greenberg's, Gurein's etc models. This paper also discusses some basic relations between traffic flow characteristics, i.e., the fundamental diagrams, speed, volume, and density relationships, hydrodynamic analogies, Traffic hump formation (shock wave) and sheds some light on the different points of view adopted by the traffic engineering community. Moving on, we review some performance indicators that allow us to assess the quality of traffic operations. A final part of this paper gives the probabilistic description of traffic flow, distribution of vehicles on a road.

1. INTRODUCTION

Traffic-flow theories seek to describe in a precise mathematical way the interactions among vehicles, drivers, and the infrastructure. The infrastructure consists of the highway system and all its operational elements, including control devices, signage, and markings. These theories are an indispensable element of all traffic models and analysis tools that are being used in the design and operation of streets and highways. The scientific study of traffic flow had its beginnings in the 1930s with the application of probability theory to the description of road traffic and with the pioneering studies conducted by Bruce D. Greenshields at the Yale Bureau of Highway Traffic on the study of models relating volume and speed and the investigation of performance of traffic at intersections. After World War II, with the tremendous increase in the use of automobiles and the expansion of the highway system, there was also a surge in the study of traffic characteristics and the development of traffic-flow theories. In December 1959, the First International Symposium on the Theory of Traffic Flow was held at the General Motors Research Laboratories in Warren, Mich.This was the first of what has become a series of triennial symposia on the theory of traffic flow and transportation. The field of traffic-flow theory and transportation has become too diffuse to be covered by any single type of meeting, and numerous other symposia and specialty conferences about a variety of traffic-related topics are held on a regular basis. Yet, even as traffic-flow theory is increasingly better understood and more easily characterized through advanced computation technology, the fundamentals are just as important today as in the early days. They form the foundation for all the theories, techniques, and procedures that are being applied in the design, operation, and development of advanced transportation systems. This elementary and brief introduction to traffic flow theory is included to extend the engineer's knowledge in this vital area and to relate the theory to other aspects of traffic engineering.

2. BASIC RELATIONSHIPS

Earlier, the major elements of traffic flow were outlined as: (1) composition or classification; (2) volume; (3) origin and destination ;(4) quality; and (5) cost. Traffic flow is concerned particularly, with three of these elements: composition, volume and quality.

One objective of traffic flow theory is to derive theoretical relationships between the various traffic variables so that the engineer can determine the characteristics of traffic streams and hence predict the consequences of alternative designs. Initial work is concentrated on the flow, density and space-mean-speed variables as these three variables are of particular interest since flow describes the how many vehicles are moving, and flow (U_s = resultant demand), density (D) and space-mean-speed (V) together describe the quality of service experienced by drivers which can serve as a useful first approximation to travel costs. These three variables are related to each other by the equation:

$$V = D * U_s \tag{1}$$

This relationship represents the fundamental equation of traffic flow. If a relationship is established between any two of the variables, the relationship of the third can be controlled by equation (1). Obviously the possible combinations are volume and density, speed and density, and speed and volume, all of which have been investigated empirically.

At this point it is useful to consider some of the characteristics of the physical driver-vehicle-highway system and the manner in which it operates, and to set out the basic conditions that must be satisfied by any explanatory theory. The definitions clearly require volume to be zero when density is zero, and requires volume equal to be zero at maximum density (i.e., when vehicles are lined up end-to-end). Further, it is evident that volume decreases before density reaches a maximum value (Note that the maximum volume that can pass through a given section of highway is referred to as the highway "capacity" by traffic engineers). The relationship between between volume and density, then, must have a general form similar to that illustrated in **Figure 1**.

This has been termed the "fundamental diagram of road traffic by Haight". Clearly there are innumerable volume-density functions that will pass through the zero-volume points and have a maximum volume value falling in between, and several functions have been proposed or derived from empirical data or theoretical consideration.

Since space mean speed is volume divided by density, the slope of the line $\overline{OA}$ in the Figure 1 represents the space mean speed corresponding to volume

V_A and density D_A . It is also clear that for the volume level V_A , there is another possible speed, defined by the slope of the line $\overline{OB}$ and corresponding to density D_B. Further, at the jam density D_j, speed is zero and as the volume and density approach zero, the speed will be equal to the means free speed (i.e., the speed drivers will assume when free or virtually free from inference by other vehicles). The mean speed will be a function of the drivers, their vehicle characteristics, the highway characteristics (lane width, sight distance, etc.), and other factors such as lighting and weather.

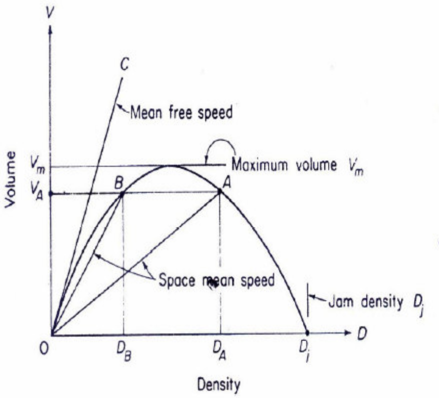

FIGURE 1. The fundamental diagram Of road traffic

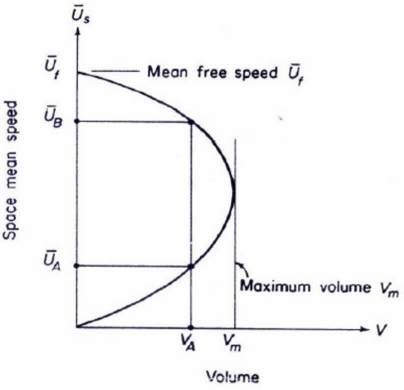

FIGURE 2. General form of volume- speed relationship

In general, the relationship between speed and volume will be of the form shown in **Figure 2**. As volume increases, the space mean speed decreases, and travel time increases. If the input volume for the facility approaches V_m, the dynamics of traffic flow and shock wave action may cause the capacity to be reduced below V_m , with speeds corresponding to lower-speed portion of the curve; in such an instance, the maximum volume that can pass through the roadway section might be reduced to V_A, say, and the speed to U_A. From the traffic engineers' and highway users' point of view, the reduction in speed and in capacity is clearly an undesirable situation, since only few vehicles can pass and since they must travel at lower speeds than appears reasonable for the section of highway. In the section that follows, the results of the more pertinent research on traffic flow theory will be summarized and applied to these sorts of problems.

3. EMPIRICAL STUDIES OF VOLUME, DENSITY, AND SPEED RELATIONSHIPS

One of the earliest recorded works is that of Greenshields in 1934. He found a linear relationship as shown in **Figure 3** between average density and average speed of the form

$$U_{s} = U_f - \left(\frac{U_f}{D_j}\right)*D \tag{2}$$

Where U_f is the mean free speed, and D_j is the jam density. Thus for a Greenshields' data with the mean free speed U_f as 46 mph and the jam density D_j as 195 vehicle per mile; thus for his data,

$$U_s = 46 - 0.236*D.$$

If a linear relationship between speed and density is assumed, then the corresponding functions for volume and density and for volume and speed are parabolic. The volume density relationship can be derived by substituting *V/D* for U_s in equation (2); the result is:

$$\mathrm{V} = \mathrm{U_f\,D} - \frac{U_f}{D_j}\mathrm{D}^2 \tag{3}$$

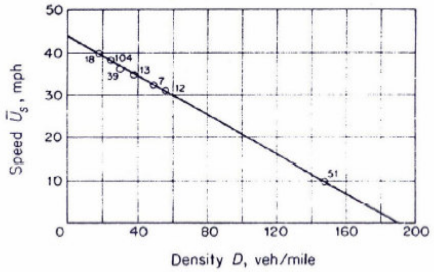

FIGURE 3. Speed-density relationship based on 244 groups of 100 vehicles curves.

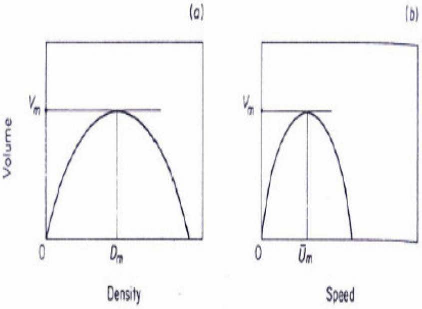

FIGURE 4. Parabolic volume-density and volume-speed

(a)Volume-density relationship; (b) Volume-speed relationship.

Similarly, by substituting $V/\,U_s$ for D in equation (2), the volume-speed relationship is:

$$V = D_j\,U_s - \frac{D_j}{U_f}\,U_s^2 \tag{4}$$

These two relationships are illustrated in **Figure 4**. To determine (1), the density, and (2) the speed at which the flow or volume is at maximum, Equations (3) and (4) must be differentiated with respect to density and speed, respectively; the resulting differentials can then be set equal to zero and solved. Thus:

1. Density when volume is maximum (D_m) :

$$\frac{dV}{dD} = U_f - \frac{U_f}{D_j} 2D = 0$$

and $$D = D_m = \frac{D_j}{2} \quad (5)$$

2. speed when volume is maximum (U_m) :

$$\frac{dV}{dUs} = D_j - \frac{D_j}{U_f} 2U_s = 0$$

and $$U_s = U_m = \frac{U_f}{2}$$

(6)

From Equations (1) , (5) and (6) ,the maximum volume is

$$V_m = D_m U_m = \frac{D_j U_f}{4} \quad (7)$$

Thus for the Greenshield's data with $U_f = 46$ mph and $D_j = 195$ veh/mile, the $D_m = 98$ veh/mile, $U_m = 23$mph, $V_m = 2{,}240$ veh/hr Greenberg analyzed data collected in the north tube of the Lincoln Tunnel and fitted an exponential function to the speed and density observations which are shown in the **Table 3.1.** His hypothesis that speed and density are related exponentially was based on a theoretical analysis in which high-density traffic flow was assumed to be analogous to continuous fluid flow. The form of the function was:

$$D = Ce^{bUs} \quad (8)$$

Where e is the base of the Napierian or natural log system. The parameters C and b can be estimated by making a least-square regression fit to the data shown in **Table 3.1.** To express Equation (8) in a linear form, algorithmic transformation is made, as follows:

$$\log_e D = \log_e C + b\, U_s$$

Putting $Y = \log_e D$, and $c = \log_e C$, the linear equation becomes

$$Y = c + b\, U_s \qquad \text{or} \qquad Y = a + b\,(x - \bar{x}) \quad (9)$$

Where x is U_s, and $\bar{x}$ is the mean value of U_s (that is, $\bar{x} = \sum \left(U_s / n \right)$).

From the data in the Table 3.1, after transformation,

$\Sigma y = 81.098$, $\Sigma U_s = 285$, $\Sigma y^2 = 368.603$ ' $\Sigma U_s{}^2 = 5{,}453$,$\Sigma U_s y = 1{,}229.325$

From the above equations, $b = -0.0581$ or $-1/17.2$ and $c = 5.425109$, thus

$$\log_e C = 5.425109, \qquad C = 227,$$

and $$D = 227e^{-(Us/17.2)}$$

(10)

The estimated coefficient of determination calculated comes out to be $r^2 = 0.988$ and the standard error of estimate for density once calculated yields s = 4.93 vehicles/mile.

TABLE 3.1 Speed and density observations in Lincoln Tunnel

Volume ,V *veh/hr*	Speed U_s, *mph*	Density D, *veh/mile*
1,088	32	34
1,232	28	44
1,325	25	53
1,380	23	60
1,480	20	74
1,558	19	82
1,496	17	88
1,504	15	94
1,410	15	94
1,344	14	96
1,339	13	102
1,344	12	112
1,188	11	108
1,290	10	129
1,188	9	132
1,112	8	139
1,120	7	160
990	6	165

Figure 5 includes the observed data and the plot of Equation (10). The exponential form gives an infinite mean free speed and a jam density of 227 vehicles/mile.

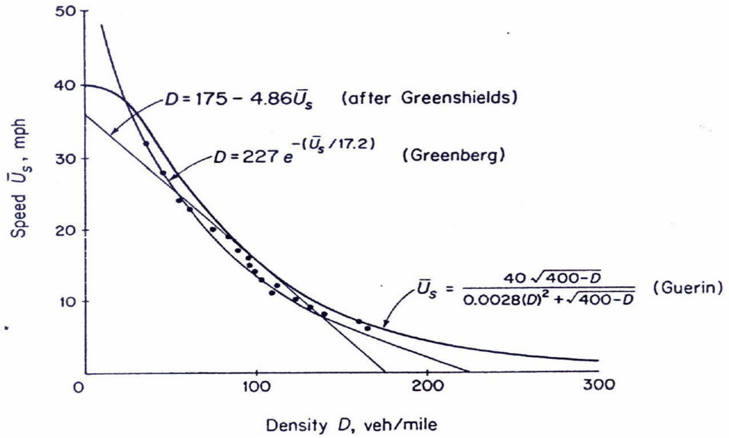

FIGURE 5 Density-speed relationships

In the former respect, the exponential violates the boundary condition for speed at zero density. The relationship between speed and volume follows directly from Equations (1) and (10):

$$V = U_s\ (227\mathrm{e}^{-(Us\ /17.2)} \qquad (11)$$

This volume-speed equation is shown in **Figure 6**, together with Greenberg's field data. The standard error of estimate and estimated coefficient of determination have been computed, giving s=61.7 vehicles/hr, and $r^2 = 0.85$, respectively. From the both statistics and Figure 6, the derived form for the volume-speed relationship is not as suitable as the equation for speed-density. In particular, Equation (11) underestimates volume in the region of maximum volume. Violation of the boundary condition at zero density is also evident, since speed is infinite at zero volume. Using Equation (11), a maximum volume of 1,460 veh/hr occurs at a speed of 17.2 mph.

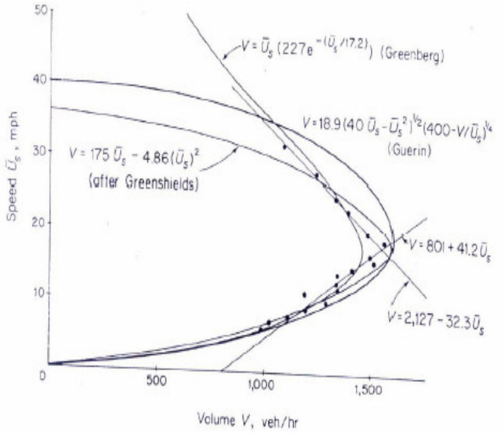

FIGURE 6 Volume-speed relationships

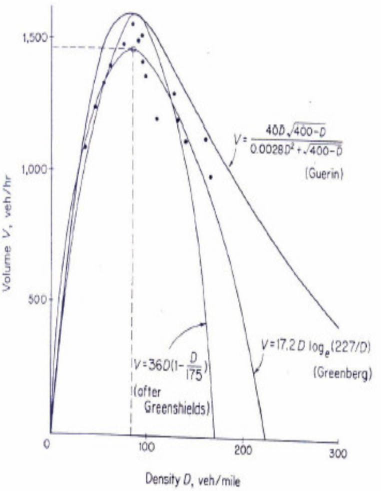

FIGURE 7 Volume-density relationships

Figure 7 shows the relationship between volume and density and the observed field data. The functional relationship is

$$V = 17.2\,D\,\log_e \frac{227}{D} \qquad (12)$$

This equation satisfies the volume-density boundary conditions and has the general form postulated for the fundamental diagram in Figure (1). The density at maximum volume, as shown in Figure 7, is approximately 82 vehicles/mile.

For purpose of comparison, a linear equation has been fitted to Greenberg's density-speed data. A least-squares regression fit was made for an equation with the following general form:

$$D = c + bU_s$$

For the data in the Table 3.1,

$\sum D$ =1,766 , $\sum U_s$ =285 , $\sum D^2$ =197.156 , $\sum U_s{}^2$ = 5,453 , $D\ U_s$ = 23,375

$$C = 175, \quad b = -4.8576$$

Thus D =175-4.86 U_s (13)

and $r^2 = 0.936$, s= 8.5 vehicles/mile

Equation (13) is shown in Figure 5, where it may be observed that the linear regression fit is not as suitable as Greenberg's equation. In particular, there are large deviations for densities less than 50 veh/mile and for densities larger than 140 veh/mile. This has resulted in a pattern of systematic errors for the regression line. However, as shown above, the r^2 value and the standard error of estimate were reasonably good for most engineering purposes.

The previous comparison illustrates an important aspect of curve fitting and a distinction should be made between a "good prediction functions" and the "true functions". If engineers are primarily interested in obtaining a "good or satisfactory prediction" equations between speed and density for same range of data values, the linear form may be preferable because it is easy to use. However, the linear form can not be assumed as the true relationship between the variables, because of having a high value of r^2 value. In fact, there is every reason to assume that it can not be the true relationship in this case since; the errors appear to be systematically distributed about the regression line, indicating that some form of concave function is closer to the true relationship. In the present case, the exponential form of the function does have a higher value of r^2 and the errors do not show any obvious bias, thus the exponential form is more likely to be a true function than a linear form, though even it may not be the true form of relationship.

Thus, as discussed previously, the linear relationship between speed and the density leads to a parabolic curve for the derived forms; they are

$$V = 175\ U_s - 4.86\ (U_s^2) \tag{14}$$

and

$$V = 36D\left(\frac{1-D}{175}\right) \tag{15}$$

It is evident all the boundary conditions specified earlier are satisfied by equations (14) and (15). The maximum volume of 1,580 veh/hr occurs at a speed of 18 mph and at a density of approximately 82 veh/mile.

Many empirical studies have been used on observations of travel time and volume with particular emphasis on developing working relationships. Guerin, in an analysis of data collected in Chicago and new Haven, suggested the use of boundary curves. For speed and density, the boundary curve is defined as:

$$U_s = \frac{U_f\sqrt{D_j - D}}{AU_fD^2 + \sqrt{Dj - D}}$$

For Greenberg's data, the following constants and parameter values are suitable:

$U_f = 40$, $D_j = 400$, $A = 0.00007$, giving

$$U_s = \frac{40\sqrt{400-D}}{0.0028D^2 + \sqrt{400-D}} \tag{16}$$

Equation (16) is plotted on Figure 5. The boundary curves for the other variables are

$$V = 18.9(40U_s - U_s^2)^{\frac{1}{2}}\left(400 - V/U_s\right)^{\frac{1}{4}} \tag{17}$$

$$V = \frac{40D\sqrt{400-D}}{0.0028D^2 + \sqrt{400-D}} \tag{18}$$

The equations, shown in Figures 6 and 7, satisfy the four boundary conditions stated previously. Guerin pointed out that the form of the fundamental diagram defined by the equation (18) also satisfies a fifth possible boundary condition, as follows:

$$\lim_{D\to 0}\frac{dU_s}{dD} = 0 \tag{19}$$

This boundary condition implies that a small number of vehicles can be added to an empty system without a decrease in the mean free speed.

A number of empirical studies have postulated a linear relationship between space mean speed and volume. One of the first can be attributed to Normann and was latter used in the *Highway Capacity Manual*. This functional form does not serve as a satisfactory basis for developing a theory of traffic flow, but it does seem to have useful engineering applications. As an illustration, linear equations have been fitted to Greenberg's speed-volume data (Table 3.1). A general equation of the following form was fitted by least squares of two groups of data:

$$V = c + bU_s$$

The first group of data included the first five entries in Table 3.1 and resulted in the following equation:

$$V = 2{,}127 - 32.3U_s \quad \text{for } U_s \geq 18.1 \text{ mph}, \tag{20}$$

$$r^2 = 0.998, \quad s = 7.5 \text{ veh/hr}$$

The remaining 13 observations in Table 3.1 resulted in the following equation:

$$V = 801 + 41.2\, U_s \quad \text{for Us} \leq 18.1 \text{ mph}, \tag{21}$$

$$r^2 = 0.939, \quad s = 41.5 \text{ veh/hr}$$

Equations (20) and (21) are shown in Figure 6. Maximum volume and the associated speed are V =1,541 veh/hr and U_s = 18.1 mph; since V is equal to $U_s D$, D at maximum volume is 85 veh/mile.

Equations (20) and (21) are only applicable for a particular range of speeds since both give unrealistic values for volume when extended beyond the range of Greenberg's data. Within that range they appear to be better predictors than any of the curves discussed earlier. Thus volume-speed curve consisting of three linear segments may prove satisfactory for many engineering problems.

3. HYDRODYNAMIC ANALOGIES

Analogies have been often drawn between the flow of fluids and the movement of vehicular traffic. However, the current evidences suggests that the equations developed from hydrodynamic analogies hold good only for high traffic densities and indicates that continuous and steady flow analogies almost totally obscure the fact that each vehicle is individually controlled. Even so, most important traffic-control problems occur only under high-density and other than "free movement" conditions; thus it might appear that better understanding of these analogies is worthwhile.

Principal contributions to this topic have been made by Greenberg, Lighthill and Whitham and Richards. To develop the basic relations, Greenberg's approach will be used. The assumptions are:

1) High-density traffic will behave like a continuous fluid and the corresponding fundamental motion equation for one-dimensional continuous fluid is

$$\frac{dU_s}{dt} = -\frac{c^2}{D}\frac{\partial D}{\partial x} \tag{22}$$

Where U_s = fluid velocity or space mean speed, mph;

D = density, vehicles/mile;

x = distance, miles,

t = time to move distance x,

c = roadway parameter

The equation (22) can be rewritten as, by expressing speed in terms of distance and time:

$$\frac{\partial U_s}{\partial t} + \frac{U_s \partial U_s}{\partial x} + \frac{c^2}{D}\frac{\partial D}{\partial x} = 0 \tag{23}$$

2) For a fluid, mass per unit time is constant (this is normally termed the continuity of flow); thus if traffic and fluid flow are analogous,

$$\frac{\partial D}{\partial t} + \frac{\partial V}{\partial x} = 0 \tag{24}$$

Where *V* is the volume or flow rate in vehicles per hour. Also equation (23) can be restated:

$$V = U_s D \tag{25}$$

3) If it is further assumed that speed is only a function of density, Equation(23) to Equation (24) may solved to yield speed in terms of density:

$$U_s = c \log_e \frac{D_j}{D} \tag{26}$$

Where D_j is the jam density. Substituting speed from equation (25) into equation (26) gives the following result:

$$V = cD \log_e \frac{D_j}{D} \tag{27}$$

Equation (27) defines the form of the fundamental diagram shown previously in Figure (1). It can easily be verified that:

1. When V =0, D =0 or D =D_j
2. When D =0, U_s =∞, D =D_j , U_s =0

Thus, as a complete theory of traffic flow, the formulation violates the boundary condition that at zero volume and density the speed is the free mean speed. This is particularly obvious in Figure (6), where volume and speed are graphed. This figure also illustrates that at high volumes (and hence densities) the hydrodynamic analogy is a good description.

Other useful concepts can be derived from fluid flow analogy to traffic flow. Lighthill and Whitham have shown that the speed of waves “carrying” continuous changes of volume through a vehicular flow is given by dV/dD or the slope of the fundamental diagram; thus

$$U_w = \frac{dV}{dD} = \frac{dU_s D}{dD} = U_s + D\frac{dU_s}{dD} \tag{28}$$

Where U_w is the speed of the wave in miles/hr.

As shown previously (e.g., Figure 5), the space mean speed of traffic decreases with increasing density, and thus dU_s/dD will be negative. Also, equation (28), the speed of the wave (U_w) will be less than the mean speed of traffic (U_s) and the wave will move backward relative to the mean vehicular flow. At low densities, when interaction between vehicles is very small, dU_s/dD approaches zero, at which point the wave velocity equals the vehicle speed. At densities above the point of maximum volume, the waves will move backward relative to the road, and under the conditions for maximum volume, the wave is stationary relative to the road; at lower densities, the wave moves forward relative to the road (see **Figure 8**).

Since the wave velocity changes with the vehicular density, it is possible to have different waves traveling through a vehicular stream. A particularly interesting case is a section of road with low-density flow immediately followed by a section with higher-density flow. This situation might be caused by an accident, a reduction in number of lanes, an entrance ramp, or other constricted circumstances. The wave in the lower-density traffic will travel forward (relative to the road) at a higher speed than the wave in the higher-density flow. When these waves “meet” , there will be a change in flow and a new wave will form. Both the original waves and new wave will move forward relative to the road. The new wave has been termed a shock wave by Lighthill and Whitham, and its speed is given by the slope of the chord joining the respective points on the fundamental diagram (**Figure 8**); analytically, its speed U_{sw} is as follows:

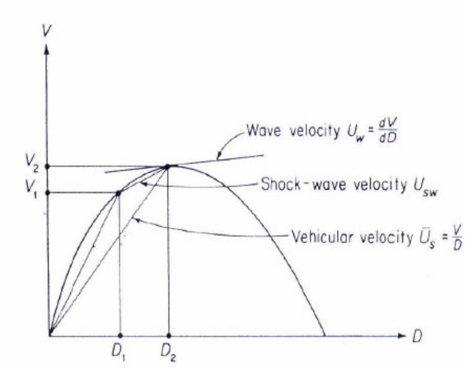

FIGURE 8 Velocity, volume and density relationships

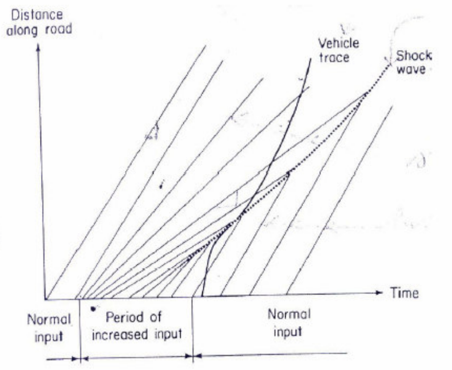

FIGURE 9. Shock wave relationships

$$U_{sw} = \frac{V_2 - V_1}{D_2 - D_1} \tag{29}$$

Lighthill and Whitham illustrate their work with several examples concerning conditions that sometimes occur in vehicular flow. The first example they tern a “traffic hump”, which is a period during which the flow of vehicles onto a highway suddenly increases. A typical example might be the release or discharge of large number of vehicles from a parking lot. The highway is assumed uniform along its length and hence the fundamental diagram will have the same form at all points. The wave pattern of flow may be illustrated conveniently on a space-time diagram as shown in **Figure 9.**

The light lines on the Figure 9 show the paths of waves, each traveling at constant speed along the highway. During periods of normal flow input, the wave velocities are the same and hence are shown by parallel lines in Figure 9. As the input flow and densities increase, the wave velocity is reduced. As the period of increased flow ends, the wave velocities increase again until they reach normal value, as shown on Figure 9. The faster waves at the right will meet the slower, causing a discontinuity and the formation of a shock wave, as shown by the dotted lines in Figure 9.

4. CAR-FOLLOWING THEORY

The hydrodynamic analogy discussed in the previous section relied on the assumption that high-density streams of vehicles behave like an incompressible fluid. In other words, it was assumed that individual drivers react to the presence of other vehicles in such a way that only the stream or total flow characteristics need be considered. The character of intervehicle relationships was not specified or necessary to analogy. Car-following theory, by contrast, deals with the intervehicle relationship and from this basis builds up a description of character of total vehicular flow.

Car-following theories have been developed by numerous researchers, but they often are associated with the work of Herman et. al. at the research laboratories of General Motors Corporation. Herman pointed out that the "follow the leader" types of problems basic to car following are the product of the psychological behavior of the drivers as they respond to certain stimuli. The first car-following model considered is one in which it is assumed that a driver will attempt to keep relative speed between his vehicle and the one immediately ahead as small as possible. The stimulus will be a change in the speed of the vehicle ahead, and the lagged response will be a change in the speed of the following vehicle, whereas the process is constrained to minimize differences in the relative velocity between the two vehicles. These ideas can be expressed in an equation of the form:

$$x''_n(t) = \lambda\left[x'_{n-1}(t-T) - x'_n(t-T)\right] \tag{30}$$

Where $x''(t)$ = acceleration of the nth vehicle at time t, or $a_n(t)$;
$x'_n(t)$ = velocity of nth vehicle at time t, or $U_n(t)$;
T= time lag or stimulus-response time of driver-car system;
λ = driver-sensitivity coefficient.
Equation (30) can be rewritten as:

$$a_n(t) = \lambda\left[U_{n-1}(t-T) - U_n(t-T)\right] \tag{31}$$

The driver-sensitivity coefficient λ is to account for the driver's awareness of and intensity of reaction to stimuli from the vehicle in front. As the vehicles get closer to one another, drivers pay more attention and hence their awareness of, and sensitivity to, the actions of the other vehicles increases. In other words, it is unlikely that λ would be constant, and in fact, experiments have shown that it is variant. Herman indicates that a good approximation to the sensitivity is a constant (λ_0) divided by the front-to-front distance between the vehicles. Thus equation (31) becomes:

$$a_n(t) = \frac{\lambda_0}{x_{n-1} - x_n}\left[U_{n-1}(t-T) - U_n(t-T)\right] \tag{32}$$

Again , T is the time lag or reaction-response time between a driver perceiving a change in the speed of the vehicle ahead and actually changing his own speed.

Equation (32) is a complex second-order differential equation, the solution of which involves the Laplace transform. By integrating equation (32), an expression relating speed and spacing or speed and density, can be obtained. Gazis et al. showed that the result is

$$U_s = \lambda_0 \log_e \frac{D_j}{D} \tag{33}$$

Where D_j = jam density,
D = density

Comparison of equation (32) with equation (33), which was derived earlier by using the fluid-flow analogy proposed by Greenberg, will show that the same form for the fundamental diagram of traffic flow has been derived from two independent approaches. However, as discussed earlier, this particular formulation does not satisfy the boundary condition of mean free speed at zero volume and density. In order to improve the car-following model in this respect, Edie has suggested a modification that expresses the driver sensitivity as a function of both vehicle speed and vehicle spacing. He assumes that the faster a driver is traveling, the greater will be his awareness of the behavior of other vehicles. The modified form of the equation (32) yields;

$$a_n(t) = \frac{\lambda_2 U_n(t-T)}{(x_{n-1}-x_n)^2}\left[U_{n-1}(t-T) - U_n(t-T)\right] \tag{34}$$

Where λ_2 is a constant. Integration of the equation (34) yields an expression relating speed and density as follows:

$$D = D_j \log_e \frac{U_f}{U_s} \tag{35}$$

Where D_j is the jam density and is equal to the $1/\lambda_2$. It should be noted that equation (35) has following properties;

When D =0: Us $=U_f$
When $U_s \to 0$: $D \to \infty$

Therefore the modified car-following equation satisfies one boundary condition at the expense of another. Edie, therefore terms equations having the form of equation (33) as congested models and equations having the form of equation (35) as uncongested models. He further suggests that these models provide the basis for the development of more complex models.

5. PROBABILISTIC DESCRIPTION OF TRAFFIC FLOW

The theoretical techniques described previously have in some respects dealt with opposite ends of the traffic flow behavior spectrum. The car-following models described and kept track of individual car movements, while the fluid models described the movement of the total vehicle population. Specifically, these two models can be regarded as deterministic flow behavior models, one at the micro-level and the other at the macro-level.

In many traffic engineering problems, however, an intermediate description of traffic, which preserves some of its discrete features and which deals with probabilistic aspects of flow behavior, can be particularly useful. Typical applications are studies of intersection control, left-turn storage areas, and other cases where engineer would like to estimate the traffic delay or queuing resulting from various control measures. A reasonably adequate intermediate description of traffic can be developed from the use of probability theory, statistics, and queuing theory.

As an initial example, consider a stretch of road free of traffic-control devices and on which the volume of vehicles is very light. An observer at one point on the road might note that over a period of time vehicles pass without being any regularity being apparent. Therefore flow may be described as random. That is, the number of vehicles arriving in any interval of time is independent of the number of the vehicles that arrived during any previous time interval. Earlier it was noted that this assumption is made for the Possion's distribution (as well as for gamma and exponential distributions). For Poisson-distributed arrival (n), the probability of exactly n vehicles arriving in any t-sec interval, is given by:

$$p(n) = \frac{\mu^n e^{-\mu}}{n!}, \text{ for n=0, 1, 2.........,}\infty \qquad (36)$$

Where n! =n (n-1) (n-2) … (2) (1),

$\lambda = V/T$ = mean rate of arrivals per unit time interval,

$\mu = \lambda t$ = average number of vehicles arriving in time t,

V =total volume of vehicles arriving during time T.

The probability of no vehicles arriving in the interval t (that is, n=0) is:

$$p(0) = \frac{\mu^0 e^{-\mu}}{0!} = e^{-\mu} = e^{-\lambda t} \qquad \text{for } t \geq 0,$$

If no vehicles arrive in time t, then there must have been a gap or time headway of at least t secs. Thus the probability of a headway h being equal to or greater than t is

$$P\ (h \geq t) = e^{-\lambda t}, \text{ for } t \geq 0, \qquad (37)$$

and the probability of a headway being less than t is

$$P\ (h < t) = 1 - e^{-\lambda t}, \text{ for } t \geq 0,$$

Equation (37) is a continuous function since t can assume all values from zero to infinity. (This cumulative probability is termed the exponential distribution). Since any volume of vehicles V will also have V gaps, the frequency of occurrence of a gap of size h in time T is given by:

$$\text{Freq (h} \geq \text{t)} = Ve^{-\lambda t}, \text{ for } t \geq 0, \qquad (38)$$

$$\text{Freq (h} < \text{t)} = V(1 - e^{-\lambda t}), \quad \text{for } t \geq 0,$$

If, for example 705 vehicles pass in a period of 2,691 secs, and if random arrivals are assumed, the probability and the number of various size gaps may be computed as follows:

$$\lambda = 705/2{,}691 = 0.262,$$

$$P\ (h \geq t) = \text{e}^{-0.262t},$$

$$\text{Freq (h} \geq \text{t)} = 705\ \text{e}^{-0.262t}$$

Since the headway or gap distribution equation, equation (37), is continuous, the probability of headway of exactly t secs is zero. However, for an interval, such as the interval from t_1 to t_2, the probability would be

$$P\ (t_2 > \text{h} \geq t_1) = e^{-\lambda t_1} - e^{-\lambda t_2}, \quad \text{for } t \geq 0, \qquad (39)$$

Thus for an example with t_1 and t_2 equal to 2 and 3 respectively.

$$P\,(3 > h \geq 2) = 0.5921 - 0.4557 = 0.1364$$

The analysis so far has been based on the assumption that arrivals for light traffic are randomly distributed over time. If this is the case, the Possions distribution can be used to predict vehicle arrivals and, in turn, the exponential distribution can be used to describe the headways. In order to test the hypothesis about random nature of light traffic, it is necessary to collect field data, to estimate the parameter value, and to analyze the empirical and theoretical data. Generally, various numerical or empirical studies have been conducted, which shows that for light traffic conditions, the Possions distribution can be used to approximately describe the flow. However, as the flow rate increases, the equation (37) becomes unacceptable, at least for the situations where statistical acceptance of order indicated is required. This conclusion only serves to confirm the previous car-following results that relationships do exist between vehicles, particulary as the traffic density increases.

Haight et al. have suggested use of the gamma or Erlang probability function. This probability function may be written as:

$$f(t) = \frac{\lambda}{(k-1)!}(\lambda t)^{k-1} e^{-\lambda t} \qquad \text{for} \quad t \geq 0, \tag{40}$$

Where k and λ are parameters of probability function and k is a positive integer. For Possion-distributed events or arrivals, the gamma probability function may be used to represent the waiting time t until the *kth* arrival or event. That is, $f(t)$ is equal to the probability of waiting time t until the *kth* arrival. The special case of the gamma probability function for k being to 1.0 is called the exponential probability function; it is:

$$f(t) = \lambda e^{-\lambda t} \qquad \text{for } t \geq 0, \tag{41}$$

Where $f(t)$ is equal to the probability of waiting time t until the 1st arrival or event.

6. USE OF CELLULAR AUTOMATA

Dhingra et. al (2004-05) contributed research work on traffic flow modeling concentrated on various areas like mid-block, intersections and ramps, and many of the works were on homogeneous traffic conditions. In developing nations like India, the traffic scenario is even more chaotic, as the element of heterogeneity is brought in by varying vehicular, human, road and environmental factors. Simulation have established as a very vital tool for the traffic analyst, in getting to know more about the traffic flow and to verify the performance of control systems. Cellular automata(CA), a new entrant to this area, introduced a concept of minimal modelling, against the complex modeling procedures being in use till then. It has proven to be computationally efficient in analysing mid-block traffic for homogeneous and heterogeneous conditions. The model is used to study the behaviour of heterogeneous traffic like lane changing, incident occurrence, and the effect of different compositon of traffic. The study of cellular automata (CA) gives a new computationally efficient model for heterogeneous traffic. However, these models can handle only uni-directional traffic at mid-block. Therefore, further study is required to explore the vehicular behaviour in bi-directional traffic at mid-block as well as intersection.

7. CONCLUSION

This paper essentially gives a gist of various theoretical approaches of traffic flow which describe the way in which vehicles move over parts of the highway system. The various empirical studies relating to volume, density and speed involving the approaches like Greenshield's, Greenberg's and Guerin's are presented. Macroscopic models like hydrodynamic analogies including contributions from Lighthill and Whitham, Richards are highlighted. Car-following models consisting of treatment of traffic flow at microscale, involving intervehicle relationship characteristics, details of individual vehicles are shown here. The microscale treatment of traffic flow is inspired by General Motors approach which is generally associated with the work of researchers Herman et al. Apart from above deterministic approaches, an intermediate description of traffic, preserving some of its discrete features and dealing with probabilistic aspects of traffic behavior is explained at length.

8. REFERENCES

1. "Definitions and Notations", *Highway Res. Board Spec. Rept.* 79, 1964.
2. B.D.Greenshields and F.M. Weids*, Statistics with Applications to Highway Traffic Analyses,*Eno Foundation for Highway Traffic Control,Saugatuck,Conn.,1952
3. Dhingra S. L , Gundaliya P. J. and Tom V. Mathew. "Cellular Automata: An approach for Traffic Flow Modelling" Accepted in START 2005, at IIT Kharagpur.
4. Dhingra S. L. , Gundaliya P. J. and Tom V. M. "Heterogeneous Traffic Flow Modelling using Cellular Automata" 10th WCTR 2004, Istanbul.
5. F.A. Haight, *Mathematical Theories of Traffic Flow*, Academic Press Inc., New York, 1963.
6. Gazis Denos C.: Traffic Science
7. Geetam Tiwari, *Traffic Flow and safety: Need for new models for Heterogeneous Traffic.*
8. H. Greenberg, "A Mathematical Analysis of Traffic Flow", Tunnel traffic Capacity Study, the Port of New York Authority, New York, 1958.
9. H.Greenberg,"An Analysis of traffic Flow", *Operation Research*, vol.7, no.1, 1959.
10. *Highway Capacity Manual*, Bureau of Public Roads, U.S. Dept. of Commerce, 1950.

11. L.C.Edie, “Car Following and Steady-State Theory for non-congested Traffic,” Tunnel Traffic Capacity Study Report VI, Port of New York Authority, New York, May, 1960.
12. Lighthill M.J. and Whitham G.B., “On Kinematic Waves; II. “A Theory of Traffic Flow and Long Crowded Roads,” *Proc. Roy. Sec.* (London), ser.A, vol.229, no.1178, 1955.
13. May, Adolf D., *Traffic Flow Fundaments*, Prentice Hall, 1990.
14. N.S. Guerin, “Travel Time Relationships” , *Quality and Theory of Traffic Flow*, Bureau of Highway Traffic, Yale University, New haven,Conn., 1961.
15. O.K. Normann, “Results of Highway Capacity Studies”,*Public Roads*,vo;.23,no. 4,1942
16. R.Herman, E. Montroll, R. Potts, and R.Rothery, “Traffic Dynamics: Analysis of Stability in Car Following,” *Operations Res*., vol. 7, 1959.
17. Richards P.I., “Shock Waves on the Highway,” *Operations Res*.,vol.4,no.1.1956